VARIABILITY OF PERFORMANCE IN THE CURVE OF WORK

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BY
JAMES D. WEINLAND, A.M.

L. C.

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy, in the Faculty of Philosophy, Columbia University

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Dissertation

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Variability of Performance in the Curve of Work

Introduction

Four kinds of variability in the curve of work may be discerned; namely,

1. Variation of performance at different stages of the work curve;
2. Variation of performance among different work curves, made under the same conditions;
3. Variation among individual muscular contractions;
4. Variation of performance in the course of training.

As each of these forms of variability has its own historical background, and as each requires its own method of treatment, this study will be presented in five sections: one for each of the above problems and a final one in which the relations among the various sub-divisions are discussed and practical implications suggested.

This investigation comprises one section of a series of studies of fatigue under the direction of A. T. Poffenberger,* involving 6 forms of muscular work and 4 forms of mental work. The muscular work with which this report deals consists of: lifting a 10-pound weight with the second finger in the standard Mosso ergograph; (2) lifting a 25-pound weight with a gripping motion of the hand similar to squeezing a dynamometer; (3) squeezing the Smedley Dynamometer; (4) lifting a weight of 12, 18, 24, 30 pounds by contracting the muscles of the upper arm and bending the arm at the elbow in a device adapted from the regular gymnasium chest weights; (5) lifting a 30-pound weight attached to the foot by bending the leg at the knee and drawing it toward the body; (6) lifting an 80-pound weight with a kind of rowing motion with the body in a sitting position, involving practically all the muscles of the body.

Ten subjects took part in the experiment and carried out a daily program of work for a period of about 6 months. Each

* A project supported by the Ernest Kempton Adams Research Fellowship, Columbia University.

subject passed through a training period on each machine and then entered upon a schedule of work which was planned so as to control conditions as far as feasible and to make possible a number of comparisons of specific data.

Each period consisted of: (1) A series of rhythmical muscular contractions, or lifts of the weight, lasting until exhaustion. The rhythm of these contractions was controlled by a metronome beating at 1-, 2-, or 3-second rate, except in a few cases where, with the metronome silent, the subject was allowed to choose his own rhythm. (2) A rest period was allowed of 5, 10, or 20 minutes. (3) Another series of muscular contractions followed similar to those before rest.

All work was graphically recorded upon a kymograph, so that each work period described above yielded a work curve, showing the height and number of individual muscular contractions and a record of the time signals produced by the metronome (See Figure 1, page 9.). The subjects were instructed to do their best at all times and to continue lifting the weight as long as possible. They were paid at a generous rate per hour and co-operated heartily in the experiment.

The height of each contraction in the work curve was measured with a millimeter rule, and the number of contractions counted. From these measurements the computations embraced in this report are derived. From the above brief description it will be clear that during each "work period" two "work curves" were made, the second curve following the first one after an intermission or rest period (See Figure 1.).†

† A more detailed description of the apparatus and technique in this series will be found in a forthcoming report of the Ernest Kempton Adams Research Fellowship.

CHAPTER I

The Variation of Performance at Different Stages of the Work Curve

As we are examining here the variability in height of stroke throughout the work curve, all factors which might affect this, with the exception of fatigue, were kept as constant as possible. It was not possible, of course, to regulate the determination with which the subject worked, except by giving standard directions.

It will be observed in glancing at the work curves in Figure 1, page 11, that there are two types of variability in height of stroke, one type including the downward trend of the curve and resulting in frequently shortening strokes, the other type an "up and down" variability where it will be seen, in comparing adjacent strokes, that some later ones are longer than some earlier ones.

These two types of variability were studied separately by two different methods and will be examined in the order in which they were named above.

Historical

Observations on variation of performance at different stages of the work curve have been made in industry and by means of exact experiment.

Experimental work has dealt with variations in both the mental and physical work curves. Of the former we may note: Starch and Ash[1] found that in a period of mental work, errors become less frequent as time goes on; but when an error does occur, it is followed immediately by other errors more and more frequently as the period of work continues. The explanation offered for this fact is that as work continues the mind and the neural processes involved become more and more insulated against distracting stimuli accompanied by a decrease in controlling the direction of mental energy. The decrease in the occasion of errors would seem to indicate the

[1] Starch, D. and Ash, I. F., "The Mental Curve of Work," Psychological Review, 24, 1917, pp. 391-402.

former, and the increase in the number of errors in immediate succession would seem to indicate the latter.

Tsuru Arai,[2] in reporting a study upon her own mental work, states:

"We note that the difference between the time taken for one example and that taken for another is greater in the second half than in the first half of the curve. This fact, together with the evidence of introspection of the subject, suggests that fatigue causes not only decrease of efficiency but also loss of the subject's control over herself."

In experimenting upon physical fatigue, Mosso,[3] with an instrument he named the "Ponometer," found that the mental effort involved in doing physical work increases steadily as the work period goes on. With the Ponometer the subject works exactly as he would with the Mosso ergograph in lifting a weight, except that after a lift of a certain number of millimeters the weight is mechanically released, and thus the load is suddenly taken from the subject's working finger. The work being completed for the stroke, the subject should reverse the motion and allow his working finger to relax. But it was found that, as the work period progresses, the subject continues his stroke farther and farther beyond the point where the weight is released. Mosso interprets this as indicating greater and greater effort, as work continues, to lift the weight a constant height. The degree of effort is measured by the degree of failure to inhibit a motion started against a resistance that is suddenly removed.

The Ponometric curve is the reverse of the ergographic curve. But if the weight of the Ponometer is raised by exciting the medial nerve directly, the Ponometric curve diminishes during the work period. This shows, according to Mosso, that there must be some fatigue to the central nervous system attending voluntary effort. The Ponometric curves, compared with the ergographic also demonstrate, he believes, that while mechanical work tends to diminish in fatigue, the nervous effort tends to increase progressively.

---

[2] Arai, Tsuru, "Mental Fatigue," Teachers College, Columbia, Contributions to Education No. 54, 1912.

[3] Ioteyko, Dr. Josepha, "La Fatigue," p. 33.

The progressive changes in the work curve, in so far as they indicate control or its lack, were studied by Ash.[4] He used a Mosso ergograph, modified so that the first and third fingers, instead of being fastened down as is usual, were attached to springs carrying, each one, a stylus, which recorded the extraneous movements of these fingers, while the middle finger, attached to the weight of the ergograph, did the work. He found the extraneous movements increased in size as the main work curve dwindled with fatigue.

He explains this by stating:

"The magnitude of the resistance in the fatiguing organ varies directly with the work accomplished; whereas, the resistance of non-working organs remains low. With various stimuli always coming in, then, fatigue means greater difficulty in forcing the response through the organ of higher resistance, and a greater tendency for the response to break out through the various channels of low resistance, resulting in sporadic movements and poor motor control."

He concludes:

"We should expect the first symptoms of fatigue to be a loss of control in directing activities and in executing movements so as to make them co-ordinate with volitions. . . . In other words, the ability to execute movements will remain after one has lost the power to control and co-ordinate them. . . . The shambling, unsteady gait of a traveler at the close of a long day's journey, in contrast to the firm and steady step at its beginning is the result of a loss of motor control rather than a depletion of energy."

From among observations made in industry a report by E. S. Bogardus[5] in the Journal of Sociology deals specifically with our problem. In examining the relationship of accidents to their time of occurence and position in the work curve, Bogardus found that, of 2,687 accidents where life and property were destroyed, 82 per cent might conceivably have been

[4] Ash, I. E., "Fatigue and its Effect upon Control, Archives of Psychology, 31.

[5] Bogardus, E. S., "The Relation of Fatigue to Industrial Accidents," American Journal of Sociology, Vol. 17, 1912, pp. 512-539.

avoided had not the participants lacked muscular control due to excessively prolonged activity. To quote:

"In general there is a rise in the number of accidents in the forenoon, a decided fall after the noon period of rest, and another rise in the afternoon hours."

Bogardus presents

"a list of 54 railway accidents (collisions and derailments in which the total number of killed and injured was 541 persons; the total damage to engines, cars and roadbed was $512,353) compiled from the reports of the Interstate Commerce Commission. In these cases the person or persons responsible for the accident had been on duty periods of time varying from 11 hours 19 minutes to 39 hours 25 minutes. In respect to the accidents tabulated, the Interstate Commerce Commission either states or directly implies that 'the men at fault had been on duty an excessive number of hours.' The implication is clear that the fatigue of overtime work was the chief cause of these accidents."

He concludes:

"Uninterrupted work is accompanied by muscular inaccuracy which increases irregularly and at a rate dependent on the rate of activity and on the relative difficulty of the given work for the given individual."

Part One

Variability, Including the Downward Trend of the Work Curve, in the Height of Strokes

Method

The work curve was divided into four sections with an equal number of strokes in each, if equally divisable by four. In curves the number of whose strokes was not divisible by four, the earlier sections were made the longer. Thus a curve of 49 strokes would be divided 13, 12, 12, 12; one with 51 strokes, 13, 13, 13, 12. The work curve, after rest (Figure No. 1), is divided 7, 7, 6, 6.

The average length of stroke in each quarter was then taken, the deviations from the average within each quarter

were found, and these were expressed as a ratio of the average. This is represented in Table I, page 12, which gives stroke-height records for Subject 9, taken on the arm ergograph with the 30-pound weight, 2-second rhythm. Data for Table I is taken from the same curves that are shown in Figure 1.

We have, then, as a basis for this investigation, a work curve broken into four parts, the strokes of these averaged and the deviations calculated in terms of ratios. All curves derived from the arm ergograph were so treated. The data were then tabulated in different ways.

This represents the total variability during work. Some of the variability is due to warming up at the beginning of work, some is due to diminishing output as the period continues, some is due probably to inaccuracy of movement unaccented by fatigue.

Notice will be called a number of times to these different causes of variability, but no attempt will be made in Part One to apportion the influence of these causes. Our problem here is merely to find in what part of the curve this total variability is the greatest, and to compare this variability under different working conditions.

Results

In Table II, page 13, are tabulated all the curves of Subject 1, arm ergograph, before rest. In this table, together

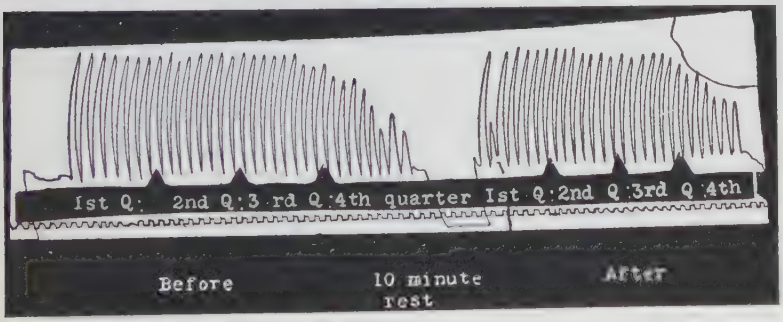

FIGURE 1

Arm ergograph curves for Subject 9, raising and lowering an 18-lb. weight, every 2 seconds. The figure shows one work period, with work before and after rest.

TABLE I

Stroke Lengths in Millimeters as Registered on the Kymograph, and Calculations of Variability for the 4 Quarters of the Curves of the Work Shown in Fig. 1.

Before Rest			*After Rest*	
		1st Quarter		
26				
26		24		
26		24		
26		25		
24	Av. 25.4	24	Av. 24	
25	AD .625	23	AD .285	
25	Ratio .024	24	Ratio .012	
25		25		
		2d Quarter		
26				
23		24		
24		24		
25		24		
24	Av. 24.4	23	Av. 23.1	
24	AD .725	22	AD .728	
24	Ratio .029	23	Ratio .031	
25		22		
		3d Quarter		
24				
25				
24				
25		23		
26	Av. 23.9	22	Av. 22.1	
23	AD 1.15	23	AD .533	
22	Ratio .048	22	Ratio .024	
22		21		
		4th Quarter		
21				
20				
21		18		
18		18		
15	Av. 15.9	15	Av. 14.2	
10	AD 4.12	12	AD 2.83	
13	Ratio .259	12	Ratio .200	
9		10		

with all others in Part One, the most striking fact is, that variability increases with fatigue. This is due largely to the fall of the curve, yet it is significant that this variability always progresses to its greatest height in the last quarter. Even though a subject starts his fourth quarter with a diminished output, his proportional reduction in output, or at least total variability, is greater there than in any previous period.

We also note in the same Table II a slightly different variability curve for the different weights. In the 30-pound curve

TABLE II

Subject 1—Relative Variability, Arm Ergograph, Different Weights, 2-Second Rhythm. All Records Before Rest.

1st Quarter	*2d Quarter*	*3d Quarter*	*4th Quarter*
		30-lb. Weight	
.021	.125	.067	.404
.030	.122	.238	.335
.025	.267	.013	.250
.017	.205	.295	.406
Av. .023	.180	.153	.349
		24-lb. Weight	
.060	.033	.239	.516
.027	.040	.149	.572
.072	.059	.082	.397
.023	.051	.117	.458
.040	.032	.312	.651
.036	.027	.183	.390
.024	.021	.265	.417
.027	.026	.301	.558
.023	.009	.068	.375
.156	.076	.147	.258
.089	.079	.129	.217
.014	.011	.236	.636
.023	.020	.365	.545
.017	.011	.014	.430
.012	.028	.115	.797
.020	.023	.043	.311
Av. .041	.034	.172	.470
		18-lb. Weight	
.011	.011	.038	.311
.016	.025	.286	.353
.022	.015	.262	.345
.031	.000	.122	.379
Av. .020	.013	.177	.347
		12-lb. Weight	
.017	.012	.037	.412
.008	.017	.025	.550
.039	.028	.037	.399
.028	.027	.296	.352
Av. .023	.021	.099	.428
		Grand Total	
Av. .033	.050	.160	.428

the variability increases steadily from quarter to quarter, while with all the lighter weights there is a fall in variability in the second period, indicating that warming-up has been accomplished and diminution of output not yet begun. This delay in rising variability, due to lessening output, is greatest with the 12-pound or lightest weight.

Table II is compiled entirely from the curves of one sub-

TABLE III

Relative Variability; Different Conditions of Rhythm.
Average of All Subjects, Arm Ergograph, Work before Rest.

	1st Quarter		*4th Quarter*	
Rhythm	*Av.*	*PE*	*Av.*	*PE*
1-second	.047	.007	.408	.039
2-second	.026	.003	.498	.018
3-second	.020	.001	.465	.019
Individual*	.021	.002	.473	.034

*Individual rhythm means that the metronome was stopped and the subject worked at his own selected pace.

ject for the different weights, all work being done at a 2-second rhythm.

But work was done under other conditions, and there were, in all, 10 subjects. In the following tables, average curves

TABLE IV

Relative Variability with Different Weights.
Average of All Subjects, Arm Ergograph, Work before Rest.

	1st Quarter		*2d Quarter*		*3d Quarter*		*4th Quarter*	
Weight	*Av.*	*PE*	*Av.*	*PE*	*Av.*	*PE*	*Av.*	*PE*
12 lb.	.021	.001	.050	.007	.107	.007	.350	.023
18 lb.	.023	.001	.032	.004	.161	.014	.385	.025
24 lb.	.028	.002	.059	.007	.145	.012	.386	.032
30 lb.	.027	.003	.100	.012	.224	.007	.320	.021

for 10 different subjects, under different conditions of work, are shown.

One factor only was changed at a time. The 2-second rhythm was used with all different weights; the 24-pound weight with all different rhythms, and different rest periods.

In Tables III and IV, the average variabilities for different conditions of rhythm and weight are given. In Table V the average variabilities in the first quarter of the curve following rest periods of different lengths will be found.

It will be noted (Table III) that the first-quarter variability is high with a 1-second rhythm, lower with a 2-second rhythm, and lowest with a 3-second rhythm. The individual rhythm (metronome silent, subject working at his own chosen rhythm) also gives a low initial variability. We might interpret this by saying that with the slow or individual rhythms adjustment is easy and first-quarter variability low.

In weight differences (Table IV), we find that with the lighter weights (12-18 pounds) first-quarter variability is low; with the heavier weights (24-30 pounds) first-quarter variability is higher.

In comparing work done after rest periods of different lengths (Table V), we find that first-quarter variability is

TABLE V

Relative Variability. Arm Ergograph, 24-lb. Weight. 2-Second Rhythm. Average of All Subjects. 1st Quarter Variability after Different Rest Periods.

Length of Rest in Minutes	5	10	20
1st Quarter Av.	.031	.038	.027
PE Av.	.004	.006	.003
No. of Cases	18	19	17

lower after a 20-minute rest than following the shorter rests. It is lower than after a 10-minute rest by a difference with a reliability equal to 87 chances in 100,[6] and lower than the variability following a 5-minute rest by a difference with a reliability equal to 79 chances in 100.

In general, the first-quarter variability rises with heavy weights, quick rhythm, or following a short rest. But the fact that among the weights the first-quarter variability of the 30-pound curve is no higher than that of the 24-pound curve; and the fact that with different rest periods, the 5-minute rest led to a lower variability in beginning the following work than did the 10-minute rest, brings a difficulty into the interpretation of the more general facts stated above.

In order to throw light on this difficulty, the standard deviations of the averages referred to were found. In the case of the weights, there were 10 average curves, one for each subject; in the case of the rest periods, about 19 individual curves, from which the SD's were calculated.

These figures are found in Table VI. Because of the number of cases, their reliability is not high; but they help to interpret the differences between the averages.

It will be observed that in the case of the weights, the SD's for the first quarter increase steadily with the increasing weights, the SD's for the 30-pound weight being higher than for the 24-pound weight.

Following different rest periods, the SD's give approxi-

[6] Garrett, H. E., Statistics in Psychology and Education, p. 134.

TABLE VI

Arm Ergograph. Standard Deviations of Variability. Average of All Records for Different Weights and Rhythms.*

	1st Quarter	*2d Quarter*	*3d Quarter*	*4th Quarter*
Different weights				
12 lb.				
Before†	.004	.034	.031	.111
After	.008	.008	.054	.043
18 lb.				
Before	.006	.018	.066	.117
After	.009	.022	.063	.096
24 lb.				
Before	.007	.031	.054	.144
After	.014	.026	.043	.156
30 lb.				
Before	.015	.058	.035	.101
After	.012	.043	.036	.158
After rest periods of different length				
5-minute				
After	.028	.036	.078	.110
10-minute				
After	.037	.053	.060	.128
20-minute				
After	.018	.051	.088	.188
All Records Averaged				
Before	.005	.024	.036	.103
After	.007	.004	.045	.093

*With changing weights the rhythm was kept constant at 2 seconds, and the rest period at 10 minutes; with changiug rest periods the rhythm was kept constant at 2 seconds, and the weight at 24 lbs.

†Before and After, refer to work done before or after the rest period.

Tables III, IV, and V state average variabilities under different conditions. The SD's of those averages for conditions stated above are given here.

mately the same results as the averages, the SD's following the 5-minute rest being lower than those following the 10-minute rest. We must assume that after a 5-minute rest, first-quarter variability is lower than after a 10, either because a balance between the loss of warming up and recovery from work is being passed through, or because of a chance difference.

We have then, as given above, the general fact that first-quarter variability, before rest, under conditions of quick rhythm and heavy load, is greater than with the reverse of these conditions, slow rhythm and light work, indicating a relationship between variability of output and conditions of work.

And we have also the fact that first-quarter variability,

TABLE VII

Records From the Arm Ergrograph. Averages for All Subjects in Relative Variability.

	1st Quarter	*2d Quarter*	*3d Quarter*	*4th Quarter*
Subject	*Before Rest*			
I	.033	.050	.160	.429
II	.025	.051	.169	.468
III	.027	.052	.174	.373
IV	.029	.047	.186	.460
V	.017	.080	.221	.357
VI	.031	.113	.236	.276
VII	.025	.079	.166	.226
VIII	.016	.022	.121	.541
IX	.026	.038	.114	.221
X	.028	.070	.158	.304
Av.	.025	.060	.170	.365
SD	.005	.024	.036	.103
PE	.003	.016	.024	.069
	After Rest			
I	.044	.068	.191	.366
II	.027	.050	.142	.462
III	.031	.077	.167	.369
IV	.028	.045	.166	.465
V	.022	.059	.225	.360
VI	.038	.093	.231	.299
VII	.036	.056	.124	.239
VIII	.021	.023	.111	.519
IX	.036	.027	.080	.219
X	.031	.055	.163	.324
Av.	.031	.055	.160	.362
SD	.007	.004	.045	.093
PE	.005	.003	.030	.063
Diff.	.006	.005	.011	.003
PE	.001	.005	.012	.029

following work and a rest, is higher than first-quarter variability when the subject is fresh. In Table VII are the averages of all work curves of all subjects, showing this difference to be reliable.

These results, stated in the two paragraphs above, would indicate very strongly the possibility of using variability as a means of measurement.

Quick rhythm, heavy weights, and insufficient rest might all be considered as involving difficulty of adjustment to work. It is possible that variability of output might measure this difficulty.

This possibility must be left as a suggestion, because in the nature of our experiment, though the subjects were instructed to do their best and work during each period to exhaustion, there was no mechanical means to ensure their doing so, and it is improbable that they always did. The variability was affected not only by the conditions named but also by the effort put into the work by the subject.

Among the results stated in regard to the difficulty of adjustment under different conditions, there is one of particular importance: it is that the first-quarter variability following work and a rest is higher than first-quarter variability when the subject is fresh; and that this initial variability of the second work curve changes with the duration of the rest. These facts suggest the possibility of measuring the amount of previous work done, or the degree of recovery from work, by a test for variability.

And variability as a measure of previous work done, or fatigue, would be valuable, both because it is evident in the first quarter of the work curve, and because variability is not very apparent to the worker and so not apt to be feigned.

Heretofore we have been noting differences in variability for the first quarter of the curve. In Tables VII and VIII it can be seen that there are differences in the other quarters, but that they do not bear any constant relation to the conditions of work nor are they as reliable as are the differences for the first quarter.

In Table VIII the differences in variability for the curves before rest, under different conditions of weight and rhythm, are compared with work under the same conditions after rest. The work before and after rest periods of different lengths is also compared.

In this table only differences with some semblance of reliability are given, and the differences are stated plus or minus, in terms of the work done after rest.

Here again it is seen that fairly reliable and constant differences are found for the first quarter of work. For the other quarters the reliablity is much less, and the differences are not so constant.

We have, then, differences in reliability in the first quarter of work that seem to bear some direct relationship to the conditions of work. In the second, third, and fourth quarters of

TABLE VIII

Relative Variability. Records From the Arm Ergograph. Differences Between Before-Rest and After-Rest Work.

Wgt. lbs.	*No. cases*	*1st Quarter*		*2d Quarter*		*3d Quarter*		*4th Quarter*	
		Diff.	*PE*	*Diff.*	*PE*	*Diff.*	*PE*	*Diff.*	*PE*
12	10	+.009	.006	....		....		....*	
18	10	†+.003	.002	+.016	.006	....		....	
24	10	+.006	.003	....		....		....	
30	10	+.005	.004	....		—.023	.009	+.049	.040
Rest Periods; Minutes		‡							
5	18	+.008	.004	+.011	.007	....		—.035	.028
10	19	+.009	.005	....		....		—.085	.031
20	17	....		+.018	.010	....		+.044	.029
Different Rhythms; Seconds									
1	10	....	....	....		....		....	
2	10	+.004	.003	....		....		....	
3	10	+.010	.003	....		....		—.061	.032
Indiv.	10	—.023	.014	....		....		+.048	.039

*Only differences with some semblance of reliability are given.
†The sign is in terms of work done after rest.
‡With changing weights the rhythm was kept constant at 2 seconds, the rest period at 10 minutes. With different rest periods the weight was 24 lbs. and the rhythm 2 seconds. With different rhythms the weight was 24 lbs. and the rest 10 minutes.

the curve, the differences in variability do not seem to bear any constant or very reliable relationship to the differences in working conditions.

In the second, third, and fourth quarters, fatigue has been added in increasing amounts, and it would appear that fatigue has caused so much variability on its own account as to hide the smaller differences in variability, characteristic of differ- working conditions—differences that were apparent when work was started.

One might sum this up by saying that fatigue is a greater factor in producing variability than any differences in previous rest, rhythm, or weight used in this experiment. This conclusion is strengthened by results found in Part Two of this chapter.

In Table IX, page 21, is a study of individual differences. The figures for each one are composite figures averaged from all the subject's records on the arm ergograph. We see here an exception to the statement of the above paragraph, that the differences in the fourth-quarter variability are seldom important. The fourth-quarter variability of Subject 7 is

higher by a reliability equal to 98.9 chances in 100 than is the fourth-quarter variability of Subject 8. This is of importance because spoilt material and accidents in industrial work occur largely at the end[7] of working spells, and a high percentage

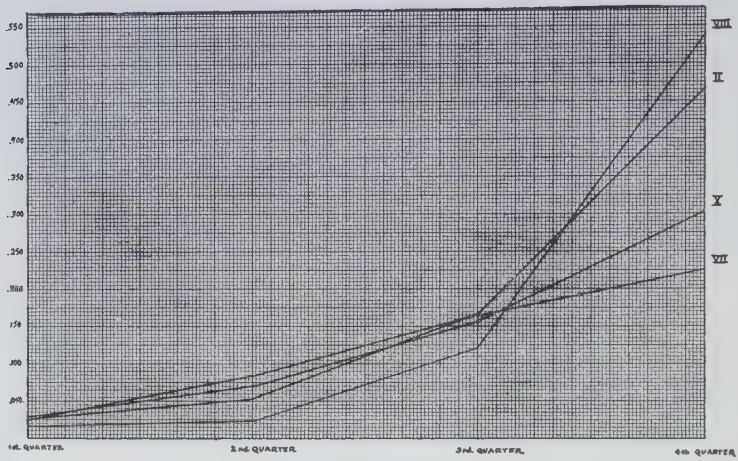

FIGURE 2

Individual Differences. Average of all arm records before rest.

of accidents can be charged to a low percentage of the workers.[8]

E. M. Neubold, in a report of the Industrial Fatigue Research Board, reports specifically on these individual differences.[9] In his conclusions he states:

"It is found that in almost all the groups the average number of accidents is much influenced by a comparatively small number of workers, and that the distributions among the workers are far from chance ones."

Or again:[10]

"The average number of accidents in any homogeneous group is much influenced by a comparatively small number of workers; and this carries with it the important practical conclusion that the elimination of comparatively few specifically

[7] Goldmark, "Fatigue and Effiicency," pp. 71-73.
[8] Vernon, H. M., "Industrial Fatigue and Efficiency," pp. 8-9.
[9] Neubold, E. M., "Industrial Fatigue Research Board," Report No. 34.
[10] Neubold, E. M., *op. cit.*, Preface, p. 14.

TABLE IX

Individual Differences in Variability. Average Curves for 4 Subjects Before Rest.

No.	*1st Quarter*	*2d Quarter*	*3d Quarter*	*4th Quarter*	*Subject*
19	.028	.070	.158	.304	X
28	.025	.051	.169	.468	II
20	.025	.079	.166	.226	VII
20	.016	.022	.121	.541	VIII

The SD of the 4th quarter average. Subject 7 is .068. The SD for the 4th quarter average, Subject 8, is .119. The difference between these averages (.315) is 2.3 times the PE of the difference, or there are 98.9 chances in a hundred of the difference being an actual one.[11]

susceptible workers from risky occupations would go far to reduce the number of industrial accidents."

From our experiment it would appear that the "specifically susceptible workers" might be picked out by some properly devised test.

Part Two

Variability from Stroke to Stroke in the Fatigue Curve, Measured in Deviations from a Smoothed Curve

Method

Part One is a study of the total variability in a work period, including both the fall of the curve and incidental deviations from this curve.

Part Two is a study of incidental deviations alone. A smoothed curve was established by the method of the sliding average, and then the deviations of the actual strokes from this smoothed curve were taken, and stated in terms of ratios of the actual strokes.

With arm-ergograph curve, Table X, pages 23-4 (Subject 6; weight, 12 pounds; rhythm, 2 seconds), a sliding average of 9 strokes was used. The first 9 strokes were totaled and averaged, and this average was set over against the fifth number; the strokes from 2 to 10 inclusive were totaled and averaged, and this average set over against the sixth number. Strokes 3 to 11 next, etc.

[11]Garrett, H. E., Statistics in Psychology and Education, p. 135.

The deviations then of the fifth smoothed from the fifth actual, and the sixth smoothed from the sixth actual, etc., were taken and stated in ratios of the fifth, sixth, etc., actual strokes.

Results

The curve, Table X, pages 23-4, is a good example of results obtained with this method. In Figure 3 the deviations are plotted exactly as found. In Figure 4, page 23, the deviations

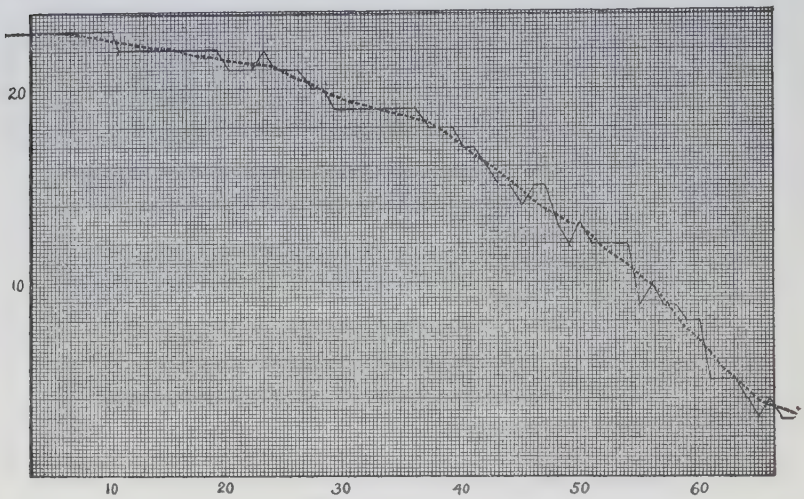

FIGURE 3

Subject VI. Arm Curve No. 46. Smoothed Curve (dotted line) and Original Curve (solid line).

are plotted in ratios, the central line representing zero deviations from the smoothed curve, and the plus and minus points representing the relative deviations of the original strokes above or below the smoothed curve. It will be noted that the relative variabilities gradually get larger and larger as the work goes on, reaching their highest values at the end.

Before conclusions can be drawn from these results, however, there are two possibilities to be considered. The first of these is that much of this greater variability at the end of the curve may be due merely to inaccuracy of movement, without fatigue; and the second, that much of the apparent variability may be due to the method used in taking the sliding averages on a falling curve.

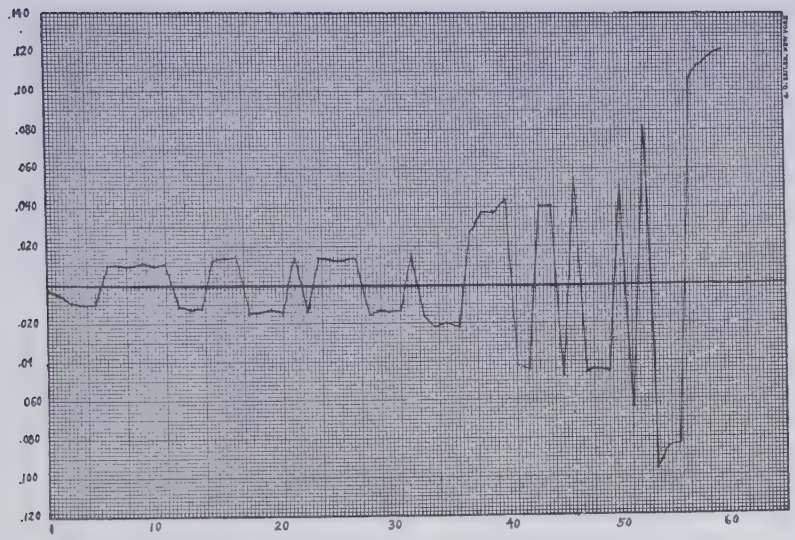

FIGURE 4

Subject VI. Arm Curve No. 46. Relative Deviations from Smoothed Curve.

TABLE X

Subject No. 6

Arm Record, 12-lb. Weight, 2-Second Rhythm. With Curve Smoothed and Differences Between the Smoothed and Original Curve Stated in Ratios of the Corresponding Strokes of the Original Curve.

Curve	*Smoothed*	*Difference*	*Relative Difference*	*Relative Difference Smoothed*
23				
23				
23				
23				
23	23	0	0	
23	23	0	0	
23	23	0	0	
23	23	0	0	
23	23	0	0	—.003
23	23	0	0	—.005
23	22.89	.11	004	—.007
23	22.79	.21	009	—.009
23	22.67	.33	014	—.010
23	22.56	.44	019	—.010
22	22.44	.44	019	+.010
22	22.33	.33	014	+.010
22	22.22	.22	009	+.010
22	22.11	.11	004	+.011
22	22.	0	000	+.010
22	21.89	.11	005	+.011
22	21.78	.22	010	—.011
22	21.67	.33	015	—.012
22	21.67	.33	015	—.012

(Continued on next page)

TABLE X—(*Continued*)

Subject No. 6

Arm Record, 12-lb. Weight, 2-Second Rhythm. With Curve Smoothed and Differences Between the Smoothed and Original Curve Stated in Ratios of the Corresponding Strokes of the Original Curve.

Curve	*Smoothed*	*Difference*	*Relative Difference*	*Relative Difference Smoothed*
21	21.56	.56	025	+.013
21	21.44	.44	020	+.014
21	21.33	.33	015	+.015
22	21.11	.11	005	—.015
21	20.89	.11	005	—.014
21	20.67	.33	015	—.013
21	20.44	.56	027	—.014
20	20.22	.22	011	+.014
20	19.89	.11	005	—.014
19	19.67	.33	016	+.014
19	19.44	.56	028	+.013
19	19.22	.22	011	+.012
19	19.11	.11	005	+.014
19	18.89	.11	005	—.015
19	18.78	.22	011	—.014
19	18.67	.33	017	—.014
19	18.44	.56	030	—.013
18	18.22	.22	012	+.016
18	17.89	.11	006	—.016
18	17.44	.56	032	—.021
17	17.00	.00	000	—.020
17	16.44	.56	034	—.022
16	16.11	.11	006	+.027
15	15.78	.78	049	+.037
15	15.22	.22	014	+.037
14	14.67	.67	045	+.044
15	14.11	.89	063	—.042
15	13.78	1.22	088	—.043
13	13.44	.44	033	+.041
12	13.11	.89	068	+.041
13	12.89	.11	008	—.047
12	12.22	.22	018	+.055
12	11.67	.33	028	—.046
12	11.22	.22	019	—.044
12	10.89	1.11	101	—.045
9	10.33	1.33	128	+.051
10	9.89	.11	011	—.064
9	9.11	.11	012	+.082
9	8.33	.67	080	—.096
8	7.56	.44	058	—.085
8	7.00	1.00	143	—.083
5	6.22	1.22	196	+.106
5	5.67	.67	134	+.144
5	5.00	.00	000	+.119
4	4.44	.44	099	+.121
3	3.89	.89	228	
4	3.67	.33	090	
3	3.44	.44	127	
3	3.22	.22	068	
3				
3				
3				
3				

Inaccuracy of Movement: The problem of the inaccuracy of movement was studied first by Fullerton and Cattell[12] who tested Weber's law in this connection, found it lacking, and annunciated a law of their own; and later by Woodworth[13] who found the results of his experiment to fall between those of Weber and the Fullerton-Cattell expectation.

According to Weber's law one would expect the error of movement to be in direct proportion to the length of stroke. Relative figures for errors, then, such as we have used, should remain fairly constant throughout the curve.

Fullerton and Cattell found the error did not increase in direct relation to the length of the stroke, but in relation to the square root of the length of stroke. The argument of Fullerton and Cattell begins with the premise that error in movement is not a quantity of sensation but an error of observation. It is mathmetical and statistical rather than psychological. To quote from Woodworth's discussion:

"If the act of perceiving a given magnitude is in any sense a combination of the acts of perceiving its parts, then the average error in the perception of the parts would be combined into a total average error; this would, however, be less than the sum of the component average errors, because the component errors would be as likely to be in opposite directions as in the same. In such a situation the theory of probability finds that the combined error should be equal to the square root of the number of units. All this applies to the variable error, not to the constant error.

"Suppose, in attempting to draw a line a decimeter long, I am subject to an average error of 5 millimeters. If now I am to draw a line 2 decimeters long, by drawing first one decimeter and then adding another, the average error of any single decimeter would evidently come in twice. But inasmuch as the errors may be either positive or negative, and if combined by mere chance may act either with or counter to each other, the average error of the double line would not be as great as 10 millimeters. Theory and experiment agree that it would be equal to the error of a single decimeter multiplied by the square root of 2. If a line 3 decimeters long be drawn in the same way, the average error of the whole would not be that of a single decimeter multiplied by the square root of 3, etc."

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[12] Fullerton and Cattell, "The Perception of Small Differences with Reference to the Extent, Force, and Time of Movement," 1892.

[13] Woodworth, R. S., "Accuracy of Voluntary Movement," Collected Writings, pp. 12-13.

TABLE XI

Inaccuracy of Movement and Variability from Fatigue.
Subject 1—Arm Record: Weight 24 lbs., Rest 10′, Work After Rest.

				Sliding Average of 5.						
1.	25									
2.	26		D				*Ratio*			
3.	26	25.6	.40	1	2	3	1	2	3	*1 minus 3 in Ratios*
4.	26	25.6	.40	D						
5.	25	25.2	.20	.32	.32	.32	.013	.013	.013	.000
6.	25	25.0	.00	.26	.32	.32	.010	.013	.013	—.003
7.	24	24.6	.60	.45	.31	.31	.018	.013	.013	.005
8.	25	26.0	1.0	.56	.32	.32	.022	.013	.013	.009
9.	24	24.4	.40	.76	.31	.31	.031	.013	.013	.018
10.	25	24.2	.80	.88	.32	.32	.035	.013	.013	.022
11.	24	23.0	1.0	.72	.31	.31	.030	.013	.013	.017
12.	23	21.8	1.2	1.00	.30	.31	.043	.013	.013	.030
13.	19	19.2	.20	1.28	.25	.28	.067	.013	.015	.052
14.	18	16.2	1.8	1.84	.23	.27	.102	.013	.015	.087
15.	12	14.2	2.2	2.24	.16	.22	.186	.013	.018	.168
16.	9	12.8	3.8	3.04	.12	.19	.338	.013	.021	.317
17.	13	9.8	3.2							
18.	12	7.8	4.2							
19.	3									
20.	2									

Arm record 31, Subject 1, is here smoothed with a sliding average of 5. Deviations from corresponding points in the original curve are then taken and these deviations are again smoothed with a sliding average of 5, giving us the first column under "1" above, which is a smoothed curve of actual deviations. Taking the first deviation of .32 from an average length stroke of 25 as a standard, under column 2 are the expectations of deviation for this curve which would be predicted from Weber's law. Under "3" are the expectations from this curve calculated from the law of inaccuracy of movement of Fullerton and Cattell.

Columns 1, 2, 3 under "Ratio" are the same as the first columns 1, 2, 3, save that they are all figured in ratios of corresponding points from the original curve

The column "1 minus 3" gives the difference in ratios in the inaccuracy that would be predicted by Fuller and Cattell for inaccuracy of movement, and that actually found under the conditions of our experiment. This difference column represents the inaccuracy that cannot be attributed to the mere fact of shorter strokes at the end of the curve.

Later Woodworth modified this statement to take account of the fact that the errors might not be combined by mere chance, but that there might be some degree of correlation between them. His statement follows:[14]

"If we combine two variables in such a way that the variations of the one are completely dependent upon the variations of the other, then for every variation of the one there will be an exactly corresponding variation of the other, and the total variation of the combination will be the arithmetical sum of

[14] Woodworth, R. S., "Professor Cattell's Psycho-Physical Contributions," Archives of Psychology, 30, p. 70.

these corresponding variations. The variability of the combinations will therefore be equal to the sum of the variabilities of the separate variables. Two variables equal to each other in average magnitude and in variability, when thus combined, will give a compound having twice the variability of either of the components; and if a magnitude is composed of several equal variables, all perfectly correlated, the variability of the compound will be proportional to the magnitude . . . a result corresponding exactly to Weber's law. If on the other hand, there is no dependence between the variations of the component variables, so that they combine by chance, then the variability will increase as the square root of the magnitude. If there should be a negative correlation between the component variables, the total variability would increase even more slowly than the square root of the magnitude; and if there were a moderate degree of positive correlation, the total variability would increase more rapidly than the square root but less rapidly than the magnitude itself. The lower the correlation between the components, the more nearly would the increase of the total variability approximate to the square root formula, and the higher the correlation; and the higher the correlation, the lower the approximation to Weber's law."

To find out just what part of the variation of the strokes in our curves might be explained by mere inaccuracy without fatigue, a curve was smoothed in the same manner as curve 46 and the deviation compared with the expectations from both Weber's law and Woodworth's statement of the Fullerton-Cattell law, postulating a zero correlation of variations. Results are given in Table XI, page 26.

Calculations of Weber's law were made in this way. Taking the first stroke (with a deviation from the smoothed curve) as a standard, the relation in which it stands to its deviation should remain constant throughout the curve. As this deviation is .013 of the stroke length (25), all other deviations should be .013 of their corresponding stroke lengths.

Calculations for the Fullerton-Cattell law were made as follows. The deviation (.32) of the first stroke is equal to the square root of the first stroke (25) multiplied by .064. All deviations, then, should be equal to the square root of the strokes they correspond to, multipled by .064.

Having found these theoretical deviations, the ratios which they were of their corresponding stroke lengths were found in the usual way.

It will be seen in Table XI, page 26, under column "1 minus

3" that the high deviations at the end of the curve cannot be attributed to mere inaccuracy of movement without fatigue.

Sliding Averages Calculated on Falling Curves: There is still the consideration that with the sliding average calculated on a falling curve the deviations between the smoothed and original curve will become greater as the curve falls more sharply, or will be due in part to the curvilinear path of the work curve.

To eliminate this factor a smoothed fatigue curve mathematically calculated by Ioteyko[15] was used to represent the typical curvilinear path of the work curve, and the sliding average method was applied to it.

The sliding average method measures the curvilinear change in the following way. If 5 points are placed at equal intervals on a straight line and their heights from a base measured, the average height of these five points will be always equal to the middle or third point, no matter at what angle the straight line deviates from the base. In other words, with the sliding average method, there would be no deviations at all, if all the points used in the calculation lay at equal intervals upon a straight line.

But if the 5 points lie on a curve, the average height of the 5 will deviate from the middle or third point, and this deviation will vary with the changes in the path of the curve.

By using the sliding average method on a smoothed curve we are then measuring changes in the curvilinear path by comparing it at overlapping distances with a straight line placed along it.

The deviations so obtained in using the Ioteyko smoothed curve may be said to represent the variability due to measuring the deviations from the curvilinear path of the work curve.

In order to make sure that the amount of variability attributed to the curvilinear path of the work curve was not a characteristic of the one Ioteyko curve selected, the same method was used on a second[16] Ioteyko curve with a slightly longer abscissa. There were small differences between the results; but as these differences were too minute to affect our

[15] Ioteyko, Dr. Josepha, "Les Lois De L'Ergographie," Planche 2, Fig. 1.

[16] Ioteyko, Dr. Josepha, "Les Lois De L'Ergographie," Planche 3, Fig. 5.

TABLE XII

Calculations for Error Due to Taking Averages on a Falling Curve, Deducted From Calculations Made on a Fatigue Curve.

Fatigue	C	* 000	—003	005	009	018	022	017	030	052	087	168	317
Ioteyko	C	005	005	004	004	007	007	009	013	014	019	032	036
Residue	V	—005	—008	001	005	011	015	008	017	038	068	136	281

Residue variability as above calculated for 3 curves.

Subject													
4	†	—019	033	—018	—018	—007	—004	013	—024	009	094	127	125
3	‡	—005	—011	005	012	009	020	057	—016	041	016	165	462
6	§	—005	—004	000	000	—003	—002	003	024	026	026	045	081

* See Table XI page 26 Column "1 minus 3"
† 24 lb. weight 2-second rhythm
‡ 24 lb. weight 2-second rhythm
§ 12 lb. weight 2-second rhythm

conclusions, the calculations for the first Ioteyko curve alone were given.

Subtracting these "curvilinear" or "Ioteyko" variabilities from those found in a raw work curve, we find a great deal of variability left unaccounted for.

In order to make this comparison as concrete as possible, we turned to arm record for Subject 1 (see Table XI, page 26), where from the stroke-to-stroke deviations calculated from a smoothed curve are subtracted the deviations possibly due to inaccuracy of movement without fatigue; and from this remainder we now subtract that variability due to the curvilinear path of the curve as described above.

In Table XII, page 29, will be found the "Ioteyko" or "curvilinear" variability placed as subtrahend and deducted from the variability remaining in our own curve for Subject 1, as described above.

It may be seen in the table that the residue variabilities increase steadily to the end of the curve.

As this work was all done on one curve, it was repeated on 3 other curves, selected at random; one each from Subjects 3, 4, and 6.

Our calculations on the Ioteyko curve were made with 20 ordinates to correspond with our own curve for Subject 1. As our curves differ in number of strokes, selected strokes were taken from which to make the subtractions from the other curves. The curve for Subject 6 is the same curve as

TABLE XIII

A Comparison of Relative Variability Curves Obtained by the Sliding Average Method. See Table IX for Muscle Stimulated by Acid (extirpated), Muscle Stimulated by Electricity (not extirpated), and Muscle Controlled Through the Nervous System.

	Extirpated muscle stimulated by carbonic acid	*Non-voluntary contractions of the flexor muscles stimulated by electricity*	*Voluntary contractions controlled by the nervous system*
1.	.024	.014	.010
2.	.021	.011	.011
3.	.018	.009	.012
4.	.018	.010	.014
5.	.017	.012	.014
6.	.013	.017	.014
7.	.011	.026	.013
8.	.009	.035	.020
9.	.022	.042	.037
10.	.023	.045	.041
11.	.023	.043	.046
12.	.045	.040	.064
13.	.044	.038	.083
14.	.028	.045	.121

that given in Table X, pages 23-4. Every fifth stroke of this smoothed (Subject 6) curve was taken. With the curves from Subject 4 and Subject 3, every other stroke was used. The method was exactly the same as that given above with the exception that the smoothed curves for our subjects were shortened to equal the Ioteyko curve by using for the final purpose of subtraction only every other, or, in the one case, every fifth stroke.

Several points on the curves of residue variability are minus quantities, due to the irregularities of the original curves and the regularity of the calculated deductions, but at the end of the curves the variability is still found to be positive and very high. This residue variability would seem to be due to fatigue. All variability that might be attributed to inaccuracy of movement alone has been subtracted, as has been the apparent variability due to the method of calculation.

Chemically and Electrically Stimulated Muscle: The question why variability increases with fatigue immediately presents itself. Some insight into this problem may be obtained by studying the results of muscles stimulated by means of electricity or acids. In this way the action of the nervous system is eliminated, enabling us to put the question whether the variability of output results from an uneven response of

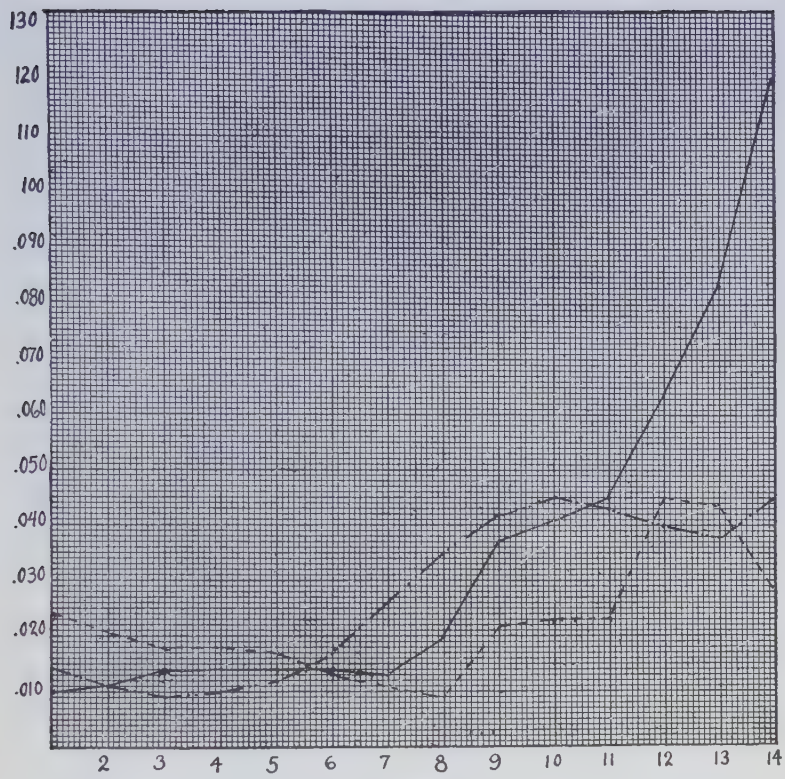

FIGURE 5

A comparison of relative deviations obtained by the sliding average method, from extirpated frog's muscle stimulated with acid, from non-voluntary contractions of the human finger stimulated by electricity and from voluntary contractions.

——————— Voluntary contractions.
—-—-—-—-—-— Non-voluntary contractions stimulated by electricity.
- - - - - - - - - - Muscle stimulated by Carbonic Acid.

the muscle due to accumulating fatigue products, or whether the cause of the variability is to be found in the nervous system. To make this comparison three curves were selected, and their coefficients of variability obtained by the sliding average method. The selected curves were (a) that of an extirpated[17] muscle stimulated by carbonic acid; (b) a curve made by the non-voluntary contractions of the flexor muscles of Dr. Maggiora's[18] fingers, stimulated directly by an electric current; (c) the arm-record curve in Table No. IX from our

[17] G. Spada, Archives Italiennes De Biologie, p. 137.
[18] A. Mosso, "Fatigue," p. 99.

own data, the subject working voluntarily. In order to make the variability curves of equal length, 14 selected strokes (the number in the shortest curve) were taken. Results are shown in Table XIII, and Figure 5.

It may be seen in Figure 5, page 31, that the voluntary contractions are much more variable in the last section of the curve than are the non-voluntary contractions. This can only be interpreted as indicating that a variable output is caused, for the most part, by the nervous system. The factor of variability must therefore be a factor in the control of the muscles rather than a factor of the fatigue products in the muscles themselves. As work continues the power of control diminishes, evidencing itself in uneven stimulation of the muscle and variable work. We conclude then that increasing variability with fatigue is due to a loss of control rather than directly to the actual exhaustion of the muscle.

Conclusions

1. Total variability, when treated in terms proportional to the work done, increases steadily from quarter to quarter of a work curve, being almost invariably highest in the last quarter.

2. First-quarter variability, before rest, under conditions of quick rhythm and heavy load, is greater than with the reverse of these conditions—slow rhythm and light work, indicating a relationship between variability of output and the difficulty of adjustment to work and suggesting that the latter might be measured by the degree of the former.

3. Higher than the first-quarter variability of output, when an individual is fresh, is that of his output following work-to-exhaustion and a rest. This initial variability changes with the duration of the rest. These facts suggest the possibility of measuring the amount of the previous work done, or the degree of recovery from work, by a test for variability.

4. The unreliability of the differences in variability in the second, third, and fourth quarters, of different curves made under different conditions, indicates an increasing number of incidental variations in work at the low output end of the curve.

5. Individual differences in variability, when the state of

near exhaustion is reached, are in some cases striking. We conclude that individuals should not be employed where variability might cause accidents without first having their tendency to a variable output measured.

6. Having deducted from the raw variability found in our curves all that variability which might seem due to inaccuracy of movement without fatigue and that due to the calculations from the curvilinear path of the fatigue curve, there still being left a variability that increases steadily throughout the fatigue curve, we conclude that variability of work increases with the oncoming of fatigue, irrespective of and aside from that variability which is part of lessening output.

7. In general, the degree of increase in variability will be affected both by conditions of work and by the individual differences of the subjects. But that increased variability is a general fact and not the peculiarity of a few curves is supported by the following evidence.

(a) It is found in the random sampling of curves given.

(b) It is in harmony with other facts of variability reported in this study. (See 1.)

(c) It agrees closely with the conclusion reached by Ash[19] who finds that control over muscular responses diminishes progressively with fatigue.

(d) It is supported by the work of Mosso who, in his studies with the Ponometer,[20] showed that inability to inhibit a movement started against resistance increases steadily toward the end of the fatigue curve. This lack of inhibitory power correlates closely with Ash's statements in regard to loss of motor control and with the results here in regard to increasing variability.

8. The increase of variability with fatigue is due rather to a loss of control than directly to the exhaustion of muscle.

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[19] Ash, I. E., "Fatigue and its Effects upon Control," pp. 15-25.

[20] Mosso, A., "Les Lois de la fatigue etudiees dans les muscles de L'homme," Archives Italiennes de Biologie, Vol. 13, 1890, p. 123.

CHAPTER II

The Deviations among Several Curves at Different Points in those Curves

In Chapter I we measured variability within the work curve, while here we will compare individual curves at different points, with average curves; and these with other average curves. The basis of all measurements will be stroke length, and the object of the study is to find whether output is more predictable and dependable in one part of the work curve than in another.

Historical

Studies of this problem have usually taken the form of investigations of daily rhythm and of the work of school children.

Some of the first studies in the field were incidental to other investigations, such as the work of Lombard[21] in 1892, who, in studying the power of voluntary contractions, found it to vary with different periods of the day. In 1900 Franz[22] wrote a paper on the current methods of estimating force of voluntary muscular contraction and on fatigue. In referring to some experiments of his own he writes:

"No series was made upon any day when the subject felt unable to do a maximum amount of work. Notwithstanding the effort to have physiological and mental conditions as constant as possible, a considerable variation was found to exist. In many cases this amounted to one quarter of the total measurement. The reason for this great variation is not evident, and the question needs investigation."

Investigations were soon forthcoming. Pillsbury[23] reported an experiment on daily variations with attention waves as the

[21] Lombard, W. P., Some Influences Affecting the Power of Voluntary Muscular Contractions, Journal of Physiology, Vol. 13, 1892, pp. 5-71.

[22] Franz, S. L., On Methods of Estimating the Force of Voluntary Muscular Contractions and on Fatigue, American Journal of Physiology, Vol. 4, 1900, pp. 348-372.

[23] Pillsbury, W. B., Attention Waves as a Means of Measuring Fatigue, American Journal of Psychology, Vol. 14, 1903, pp. 277-288.

means of measurement. In regard to his subjects he concludes that "B" and "W" are typical morning workers. Both show a continuous increase up to the evening curve, with a decrease in that. "H" has 2 maxima and 2 minima each day, so that his morning and his early afternoon records are alike, and his noon and his evening records are alike. "G" is entirely irregular. His capacity varies differently for different days, and in the average it would seem that his morning and evening conditions are very much alike.

Daily variations proved an interesting theme and a number of studies upon it were reported. T. A. Storey,[24] in working with himself, 24 years old, as a subject, found, in studying the ergographic contractions of his middle finger, that there was no harmony between the daily variation in the pulse rate and the daily variation in voluntary muscular power. There was apparently a daily variation in the power of voluntary muscular contraction that consisted of a morning rise in power lasting till 10 or 11 o'clock, then a fall lasting till noon; an afternoon rise until the neighborhood of 4 o'clock; and then a fall in power which was evident at 6 in the evening.

F. L. Wells,[25] using a typewriter especially arranged for recording results, found that an unusual amount of work during the morning makes the noon efficiency lower in both speed and accuracy, with a more deleterious effect upon the accuracy than on speed.

In 1916 Gates[26] reported a very comprehensive study of variations in efficiency during the day, with a general survey of the field. He remarks in part:

"This investigation and the examination of data from other sources indicates a somewhat unsteady yet definite diurnal rhythm of efficiency for a variety of mental and physical functions. This is close to the conclusion arrived at by Marsh when he expressed himself as favorable to the idea of a few broad daily variations of a rather general nature, co-existing with more numerous and intensive variations in individuals.

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[24] Storey, T. A., "The Daily Variation in the Power of Voluntary Muscular Contraction," American Physical Education Review, Vol. 7, 1900, pp. 188-212.

[25] Wells, F. L., "On the Psycho-motor Mechanism of Typewriting," American Journal of Physiology, Vol. 27, 1916, pp. 47-71.

[26] Gates, A. L., "Variations in Efficiency during the Day; together with Practice Effects, Sex Differences and Correlations," University of California Publications in Psychology, Vol. 2, 1916, pp. 1-156.

"Apparently all forms of motor, mental, and vital activities are subject to daily ebb and flow similar to the one we have found for mental activities in the case of school children.

"The curves of efficiency for all forms of activity are in most respects alike; in fact, as far as the morning is concerned, they are identical. But with regard to the afternoon hours, certain characteristic differences were found in individuals, it seems reasonable to suppose, which are caused either by a difference in the form and nature of the process of adaptation, by differences in the effectiveness in the assault of the various factors . . . fatigue, ennui, meals, etc."

H. L. Hollingworth,[27] making a distinction between essentially nervous rhythms and variations in efficiency due to having done previous work, found under the conditions of his experiment that the changes in the work curve were not due to organic rhythms or diurnal factors, since the same curves were secured whether the work started at 7:30 or 10:30. He finds them due rather to the influence of continuous work. This influence varies with the kind of work done.

Different in type from the studies mentioned, but bearing on the same general problem, is the mathemetical work of Dr. J. Ioteyko[28] in calculating typical fatigue curves. She writes, referring to a statement of Mosso, that the fatigue curve is often in the shape of an S with two convexities; that the form of the fatigue curve is subject to individual peculiarities and to the conditions under which the work was done. It is probable, she writes, that certain characteristics hold for race, sex, right and left of the body, etc. She also seeks to determine the effects of drugs by comparing fatigue curves.

Method

We have taken for this study measurements of the same fatigue curves as in other parts of this investigation, using the data to obtain deviations at different points in the curves.

In comparing curves some basis must be obtained for the comparison. As our fatigue curves were all of different lengths, it would be impossible to compare first strokes with first strokes, second with second, etc., since some curves have

[27] Hollingworth, H. L., "Variations in Efficiency during the Working Day," Psychological Review, Vol. 21, No. 6, 1914.

[28] Ioteyko, Dr. Josepha, "Les Lois De L'Ergographie," p. 369.

30 strokes and some 60, 63, or more. A direct comparison would contrast the middle of one curve with the end of another. In order to avoid this difficulty, the curves were divided into fifteenths, a purely arbitrary number, chosen because 15 points would not be too cumbersome to handle and yet give an adequate representation of the curve.

The method of making this division was as follows: The number of strokes in the curves were divided by 14, and these strokes recorded. Then, that the first stroke might always be represented, it was added to the selected points. In the case of a curve the number of strokes of which was not evenly divisible by 14, the intervals at the beginning of the curve, where there would be less variability, were made the longer. A curve with 47 strokes, for example, would be divided as follows: 47 divided by 14 equals 3 with a remainder of 5. The first 5 of the selected points, then, would be separated by an interval of 4, and the remainder of the points by 3. To these 14 points, the first one was added, representing the curve by 15 selected strokes. The first stroke of the curve was thus both counted separately and included in the first fifteenth; but as this had the effect of shortening the interval between the first and second fifteenth by only one stroke, and insured that the first stroke would always be among the selected fifteen, it was done.

Having the curves divided into fifteenths, average curves[29] could be obtained for the different weights, as in Table XIV and XV, and for the different individuals. Deviations are stated in ratios of the average curves in Table XVI.

The data were taken from both the arm ergograph and the Smedley Dynamometer.

Results

Comparing 8 different curves of Subject 10, working with the 12-pound weight (Table XIV, page 39), the average deviations are found to increase gradually through the first fifteenths of the work curves and then gradually to diminish as exhaustion is approximated. A work unit in one curve is more like the corresponding work unit in another curve of Subject 10 when he is so tired that he is on the point of stop-

[29] Our curves of Average Deviations are derived from 15 point curves of actual stroke lengths, calculated by Geo. M. Michaels.

ping, than it is when fatigue is reducing his output, but still he can work on.

By referring to Figure 6 below, a better insight may be gained into the differences between the curves.

It will be observed that in the first half all the curves are a good deal alike, while in the second half all the curves deviate a good deal from the average, but that one in particular deviates more than any other.

When fresh, then, regulated somewhat by the limitations set by the ergograph, the subject does about the same standard

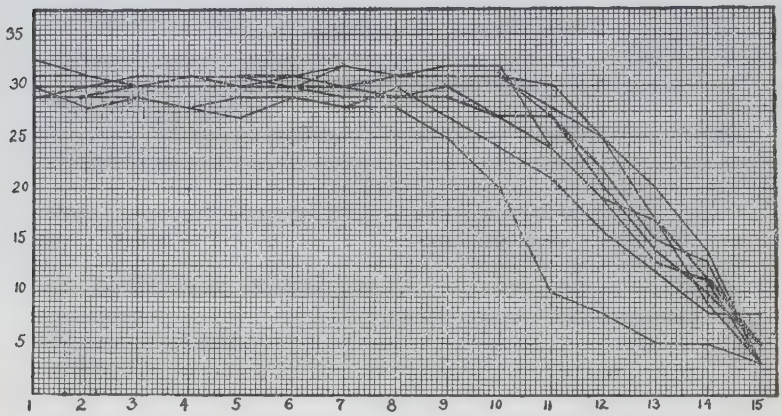

FIGURE 6

15 Point Arm Ergograph Curves with the 12-lb. Weight. Subject X.

of work on each trial. When fatigue sets in, however, and output falls enough below the limiting conditions so that work no longer is regulated by the machine, deviations commence to increase.

These deviations keep on increasing till near-exhaustion brings output again to nearly the same amount in each work curve, at a very low level of production.

The one curve which represents the greatest deviations from the average (Figure 6) would also seem to indicate low vitality for the day, and low determination. The output is slightly below the average from the very start, and when the fall in the curve begins the eighth fifteenth from the start of work, diminution in output is very rapid till the eleventh fifteen point. Here output begins to diminish less rapidly, as if the subject found that he had given up too easily, even for a bad

TABLE XIV

Subject 10—15-Point Curves, Arm Ergograph, Weight 12 lbs. Average Deviations, Actual, and in Relative Terms.

	12 Lb. Weight Curves								*Av.*	*AD*	*Rel. Diff.*
1/15	31	30	33	29	29	30	29	29	30	1.00	.033
2/15	31	30	31	30	29	28	29	30	29.7	.85	.028
3/15	30	31	30	30	29	29	30	30	29.9	.42	.014
4/15	31	31	31	31	28	28	30	30	30	1.00	.033
5/15	31	30	31	31	29	27	30	30	29.9	.92	.031
6/15	31	31	30	31	29	29	30	30	30.1	.77	.025
7/15	30	30	32	32	28	28	29	30	29.9	1.15	.038
8/15	31	31	31	31	28	30	29	29	30	1.00	.033
9/15	31	31	32	31	25	27	30	29	29.5	1.87	.063
10/15	31	31	32	31	20	24	27	27	27.9	3.37	.121
11/15	30	27	24	28	10	21	27	24	23.9	4.17	.174
12/15	25	22	19	25	8	16	20	19	19.2	3.50	.182
13/15	17	15	13	20	5	12	14	17	14.1	3.12	.221
14/15	9	13	11	14	5	8	10	11	10.1	2.12	.209
15/15	3	3	3	3	3	8	5	5	4.1	1.40	.341

day, and picked up courage to work on toward the exhaustion point.

The implications of this curve are the same as for the other curves graphed, except that its deviations are more extreme, and raise the question, which it is impossible to settle here, whether work such as this, when the individual is at a low level of vitality, may not be too costly to be worth while.

If these deviations from the average which we have been examining are stated in relative differences, they do not decrease with exhaustion, as stated above, but increase to the very end of work. It is approximately 10 times harder to tell the ratio of his average output, more or less, which Subject 10 will do on a particular day in the near-exhausted state, than it is to make the same calculation for him when fresh.

It will be observed that these results correspond in general with conclusions from Chapter I. There it was found that relative deviations in the same work curve keep on increasing to the end of work; here it is evident that relative deviations between different work curves keep on increasing to the end of work.

In Table XV, page 41, the average deviations between work curves are given for all subjects for the different weights. These curves were obtained by taking the average curve for each subject, under the different loads, averaging these average curves, and taking their deviations. It will be noted that

with the weight, ergograph, and loads used, the deviations in beginning work are much the same; but that the curves for the deviations vary with the different weights. Figure 7 helps to make evident exactly what happens. The average deviations for the 30-pound curves begin to increase soonest, and then in order follow increasing deviations for the 24-, 18-, and 12-pound curves.

A reference to the separate curves for Subject 10 (Figure 6), may help to explain this. There it was found that as soon as output fell so that there was no regulating control from the

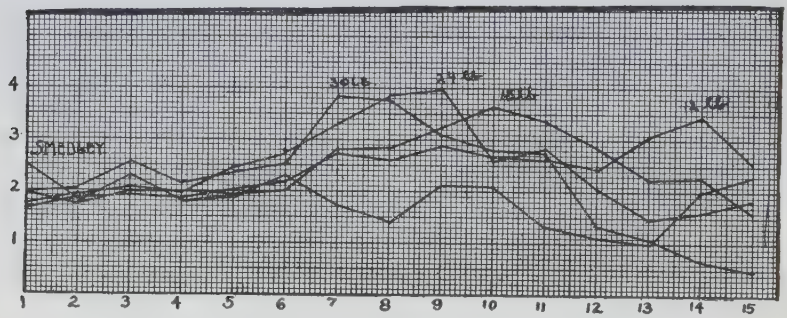

FIGURE 7
Deviations from the Average Curve.
15 Point Arm Ergograph Curves with Different Weights. Average Records of all Subjects.

ergograph, deviations began to increase; and that these deviations continued to increase until there was a leveling effect from approaching exhaustion.

Here, probably the same causes are at work. Naturally the subjects fatigue most quickly when pulling the 30-pound or heaviest weight. As fatigue begins and output falls below the regulating effect of the ergograph, the deviations among the subjects rise. Continued work brings the approach of exhaustion, and this tends to bring the subjects work back again to a general level of output. Both beginning fatigue and near-exhaustion would appear to come soonest with the 30-pound weight, and then with the 24-, 18-, and 12-pound weights in order.

It will be noted that the leveling effect of near-exhaustion is in general proportional to the load lifted. This does not indicate that the exhaustion is greater in one case than in the

TABLE XV

Average Deviations

Based on 15 Point Curves, Arm Ergograph, Different Weights. Average Curves of All Subjects. And on Smedley Dynamometer Curves for 7 Subjects.

	12 lb.	*18 lb.*	*24 lb.*	*30 lb.*	*Smedley*
1/15	1.61	1.96	1.76	1.99	2.50
2/15	1.89	1.74	1.97	2.06	1.80
3/15	1.92	1.90	2.03	2.58	2.30
4/15	1.89	1.98	2.00	2.14	1.80
5/15	2.03	1.90	2.47	2.39	1.90
6/15	2.14	2.02	2.73	2.52	2.30
7/15	2.71	2.73	3.31	3.82	1.70
8/15	2.61	2.83	3.88	3.77	1.40
9/15	2.89	3.24	3.96	3.09	2.10
10/15	2.69	3.67	2.60	2.80	2.10
11/15	2.63	3.39	2.84	2.76	1.30
12/15	2.41	2.83	2.03	1.33	1.10
13/15	3.06	2.27	1.49	1.08	1.00
14/15	3.47	2.26	1.60	.66	2.00
15/15	2.56	1.58	1.81	.47	2.30

Deviations are of each subject, at each 15th, from the average of all subjects.

other, but that with approaching exhaustion the heavier weights level the output of different subjects more than do the lighter weights. Small differences of strength can be exerted upon the lighter weights that are insufficient to move the heavier ones.

Work with the Smedley Dynamometer produced a curve with different characteristics than those of any of the weight-lift curves. With the Smedley, there is no definite lift to make and the subject sets his own standards in proportion to his own strength. With the Smedley curve (Figure 7), the deviations in beginning work were greater than with the heaviest weight among the weight-lift curves. But with the Smedley, the deviations fall soon after initial adjustment, remaining comparatively low, to rise again toward the end. Smedley deviations are the only ones to show rising deviation at the very end of the curve. The reason, again, is apparently that there was no exact standard of work, such as a weight to lift, which might make the subjects somewhat alike in their output, and as fatigue comes on they deviate more than they did through the middle section of their curves.

This increase in deviation at the end of the Smedley curves might be attributed to loss of control or perhaps to different degrees of determination among the subjects, as exhaustion approaches.

TABLE XVI

Average Deviations in Ratio of the Average Curves. Based on 15-Point Curves, Arm Ergograph, Different Weights. Average Records of all Subjects Used; And Smedley Dynamometer Curves for 7 Subjects.

	12 lb.	*18 lb.*	*24 lb.*	*30 lb.*	*Smedley*
1/15	.061	.073	.067	.074	.097
2/15	.072	.065	.074	.077	.078
3/15	.073	.073	.077	.096	.111
4/15	.073	.077	.077	.082	.093
5/15	.079	.074	.096	.102	.099
6/15	.085	.080	.109	.111	.130
7/15	.106	.104	.139	.202	.100
8/15	.108	.120	.177	.230	.084
9/15	.123	.150	.205	.228	.130
10/15	.125	.191	.157	.256	.140
11/15	.144	.212	.217	.321	.097
12/15	.159	.230	.201	.218	.089
13/15	.252	.252	.206	.230	.085
14/15	.398	.383	.308	.183	.210
15/15	.640	.544	.565	.247	.310

In regard to relative deviations, Table XVI and Figure 8, which include both weight and Smedley curves, we find them all to increase to the very end of work. The 30-pound curve is an exception in that the highest peak occurs before the end, at the twelfth fifteen point. It would appear that relative differences in output among different subjects increase progressively with continued work, unless the work is so difficult that it levels out individual differences by making all subjects, as they become exhausted, equally unable to give evidences of the small differences in strength which they may have.

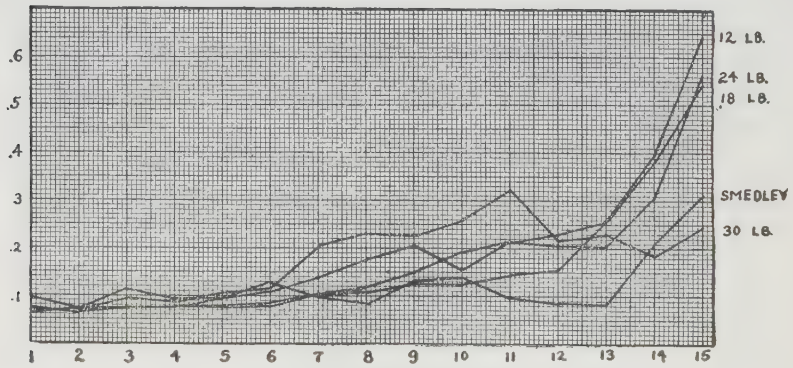

FIGURE 8

Relative Deviations from the Average Curve.
15 Point Arm Ergograph Curves; Different Weights.

Conclusions

1. Deviations among work curves at different points in those curves are found greater toward the low-output end of the curves, but less again toward the exhaustion point.

2. The amount of deviations among work curves and the points in those curves where the deviations will be greatest are dependent upon conditions of work, such as the nature and the amount of the load.

3. Relative deviations among work curves increase steadily as work is continued, except where the work is so difficult that it levels out individual differences by making subjects unable to give evidence, when fatigued, of the small differences in strength which they may have.

CHAPTER III

Variations among Individual Muscular Contractions

In this study we will look not only for variations in height of stroke but for possible variations in the way the strokes were made. We will examine variations in the length of time given to the strokes, the relation of this time to the height of the stroke and the degree of fatigue, and the relation which the time of contraction bears to the time of relaxation, throughout the different strokes of the work curve. We will also look for any idiosyncrasies in manner of working which we may find.

Historical

Individual muscular contractions have been studied principally in excised muscle, by physiologists. As this work does not bear directly on our own we may take here a summary of experimental findings in this field from the texts of Burton-Opitz.[30]

It is found that muscles do not respond precisely when stimulated, but a little later, the time elapsing between the moment of stimulation and the response constituting the latent period. This period with the gastrocnemius muscle of a frog lasts about .01 of a second; its period of contraction .04, and its period of relaxation .05.

If stimuli are repeated, contractions become mounted upon each other, till if 20 to 50 stimuli are given per second, tetanus results. If this is kept up, fatigue gradually lowers the tetanic curve till, when continued long enough, complete relaxation ensues.

Even such a brief muscular response as is required to close the eyelids is tetanic in its nature. Thus, if a certain muscle is to be contracted, the corresponding motor nerve cells send out rhythmic discharges which do not cease flowing until the contraction is to be discontinued.

Various conditions are found to affect the contraction of muscles in the following ways:

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[30] Burton-Opitz, "Elementary Manual of Physiology," Chap. 7 & 8. Burton-Opitz, "A Text Book of Physiology," Chap. 7.

1. In general, the height of the contraction is proportional to the strength of the stimulus.

2. A lasting stimulus is, in general, a more efficient exciting agent than a brief one, but this relationship holds true only as long as its duration is not unduly prolonged.

3. Increasing weights gradually reduce the height of muscular contractions; a light load, however, exerts a more favorable influence than none, because it augments the elastic tension of the contractile elements.

4. Warmth increases the amplitude of the contractions, whereas cold decreases it. The optimum temperature for the muscles of warm-blooded animals is about 37 degrees centigrade.

5. If an excised muscle is stimulated a great many times, the height of the successive contractions gradually decreases, until the writing lever remains finally in a horizontal position. The duration of the individual contractions, however, is materially increased.

6. Muscle tissue differs in its speed of reaction, the difference being dependent upon the chemico-physical peculiarities of its substance. Red muscle is a more concentrated tissue than pale muscle and embraces a larger amount of sarcoplastic material. It seems, therefore, that the greater water content of the latter exerts a favorable influence upon its rapidity of action. The same holds true in the case of nonstriated and striated muscle. Inasmuch as the latter contains more water and a smaller amount of undifferentiated sarcoplasm, its speed of contraction is much greater.

7. Certain chemicals affect the irritability and contractility of striated muscle in a more peculiar manner. Veratrin greatly prolongs its phase of relaxation. Potassium salts act as depressants, whereas sodium and calcium salts possess an excitory action. A solution of 0.7 per cent sodium chlorid is employed to keep the muscle in the best possible condition, because it is isotonic to its substance. Common stimulants of cardiac muscle are caffein and strychnin. Adrenalin diminishes muscular fatigue, owing to its stimulating action upon the circulation.

Turning to investigations with the ergograph and human muscle under normal working conditions, we find little work done in the analysis of individual contractions. One study

directly on this problem, however, is that of Schutz and Wittman,[31] who arranged a Mosso ergograph so that records were taken in the usual fashion and also directly from the weight. They used a fast running kymograph that expanded the graphic record of the strokes so that each contraction could be individually studied.

It was the theory of these men that most ergographic curves are faulty in that they show work in one aspect only—height of contraction. Records should include the time during which work was being done.

Upon measuring the time of strokes they found much fluctuation, the lift of one stroke being made in one-tenth of a second and of the next in one-fifteenth. This difference in the intensity of work, differing at times to an extent of 30 per cent or more, they held to be one of the major causes of fatigue, and not to be overlooked in considering just the objective time of the whole period.

They also found in comparing the curves written by the finger with those written directly from the weight, that 2 other factors, the stretching of the gut string attached to the weight and the vibration or jangling of the weight at the top and bottom of strokes, were important factors. The weight used with their apparatus would vibrate after a stroke from 4 to 5 seconds. When strokes were made every second, the weight, vibrating from the previous stroke, would sometimes be caught for the following stroke on an up-vibration and at other times on a down-vibration. This causes strokes of the same height to represent different values in work required to produce them.

The expanding and contracting of the gut string was also found to be a factor of importance, sometimes misrepresenting the length of a stroke by 1 or 2 millimeters.

These various factors accumulate, they maintain, and serve to emphasize the necessity of a closer investigation of the ergographic curve, including in the study the duration and timing of the strokes as well as the one factor of height.

Some work was done on this problem by Woodworth,[32] when in his studies on the accuracy of voluntary control he made

[31] Schutz and Wittman, "Zur quantitativen Auswertung der Ergogramme," Archiv fur die gesamte Psychologie, Vol. 36, 1917, p. 461.

[32] Woodworth, R. S., "Accuracy of Voluntary Control," Collected Writings, p. 57.

some speeded-drum records such as those described above, and he states in regard to the expanded strokes, "The most striking difference between these tracings is that between the sharpness of their tops . . . ." In this experiment Woodworth concluded that the blunt top was an expression of the extreme slowness at the end of the stroke which had the desirable effect of making possible finer adjustments of length.

Later[33] he considered that if blunt top strokes were habitual or were made generally in ergographic work, they would represent an unnecessary retardation in beginning relaxation and that this useless work done in holding the weight at the end of the stroke would probably hasten fatigue. He arranged an experiment on the Cattell Spring ergograph in which the subject was told to pay particular attention to relaxation of his flexion muscles by contracting the extension muscles after each stroke.

"The result was remarkable; more work was done and fatigue came less quickly. In one series of 350 contractions continuing during 45 minutes, the force exerted in the last 100 was 92 per cent of that of the first 100."

Method

In order to examine the individual strokes of a fatigue curve, a 2-drum belt kymograph was arranged and run at high speed. The individual strokes were thus expanded so that they could be measured and compared. The curves were made on a Mosso ergograph with a weight of 9.918 pounds. A one-second time interval was used, the time line being written by the metronome. The second-line of the metronome was divided into tenths, for selected strokes, by measurement, so that the time of beginning the stroke, amount of contraction time, etc., could be read in tenths of a second.

In order to limit somewhat the number of individual strokes that should be measured, 15 selected strokes were first taken from each curve by the method described in Chapter II. As these 15 strokes still made an unnecessarily large number to accurately measure, the first, fourth, seventh, tenth, thirteenth, and fifteenth were chosen and measured for duration of contraction, relaxation, and timing with the

[33] Woodworth, R. S., "Le Mouvement," p. 383.

TABLE XVII

Representing the Presence or Absence of a "Shoulder" on the 15 Selected Strokes of 8 Fatigue Curves. A Plus Mark Indicates the Shoulder Present, a Minus Mark Indicates its Absence.

Curve No.	1/15	2/15	3/15	4/15	5/15	6/15	7/15	8/15	9/15	10/15	11/15	12/15	13/15	14/15	15/15
1	—	+	+	+	+	+	+	+	+	+	+	+	+	—	—
2	+	+	+	+	+	+	+	—	—	+	—	—	—	—	—
3	—	+	+	+	+	+	—	+	+	+	—	—	—	+	—
4	—	+	+	+	—	+	+	+	+	—	—	—	+	+	—
5	+	+	+	+	+	+	+	+	+	+	+	+	—	+	—
6	—	+	+	+	+	+	+	+	+	+	+	+	—	—	—
7	+	+	+	+	+	+	+	—	+	—	—	+	—	—	—
8	—	—	+	+	+	+	+	+	+	—	+	+	+	—	—
	3/8	7/8	8/8	8/8	7/8	8/8	7/8	6/8	7/8	5/8	4/8	5/8	3/8	3/8	0

metronome. This process was repeated with eight fatigue curves and the results averaged; they will be found in Table XVIII and Table XIX.

The work was done by Subject 3.

Results

1. One striking characteristic of the expanded stroke is that the peak of the curve is usually set toward the contraction half of the stroke. One might say the stroke comes to a peak and falls off with a shoulder on the relaxation side. Stroke 4, Figure 9, is a good example (See page 49.).

Table XVII, above, gives a quantitative count of these peak and shoulder combinations on the 15 selected points in the curves. The table must be loosely interpreted because no quantitative measure could be taken of the size or position of the shoulder. Sometimes it was small, sometimes it was far down the relaxation side of the curve. In the table a plus mark was made for each stroke which had, in the opinion of the observer, a shoulder such as that in stroke 4, Figure 9.

These shoulders appear to be fairly constant phenomena though absent sometimes in the beginning strokes of a work period and always in the last strokes.

It is impossible to be sure in the case of our records whether these shoulders are due to vibration of the weight, as would be suggested by the work of Schutz and Wittman,[34] or whether

[34] Schutz and Wittman, "Zur quantitativen Auswertung der Ergogramme," Archiv fur die gesamte Psychol., 1917, 36, 461.

they are due to an unnecessary slowness of relaxation such as Woodworth discusses.[35] Their cessation, when the strokes become short at the end of the curve, would indicate vibration; but the fact that in strokes of the same height some have shoulders and some do not would indicate slow relaxation. Other evidences pointing to slow relaxation are that relaxation times are longer throughout the whole length of the work-spell than contraction times (See Table XIX, page 51.).

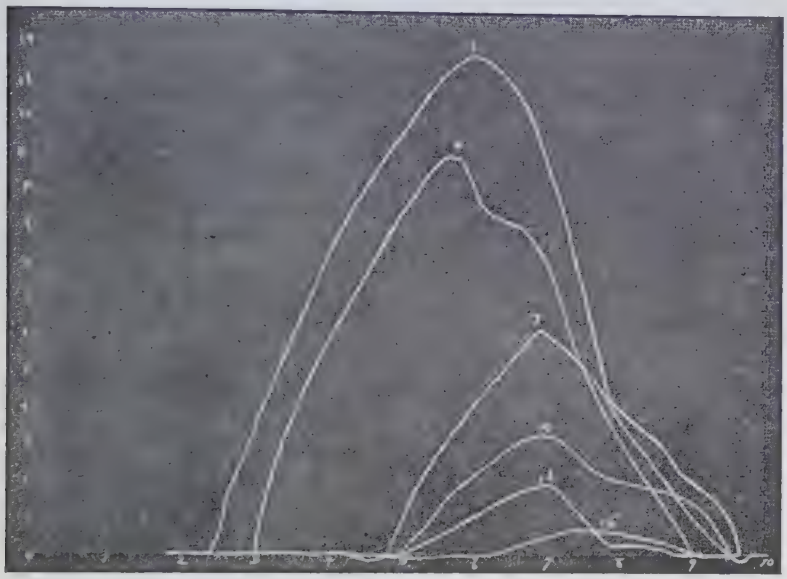

FIGURE 9
Selected Strokes from a Speeded Kymograph Record of a Work Curve. Subject III.

Moreover the shoulders are not always in the same place in relation to the peak of the curve, as can be seen in Figure 9, above and as would be expected if they were produced by a purely mechanical cause.

This problem is worthy of further investigation and measurement with an apparatus designed either to prevent vibration of the weight or to measure its extent.

2. The subject in working was instructed to keep time to a metronome. He was not able to do so throughout his com-

[35] Woodworth, R. S., "Le Mouvement," p. 383.

plete work curve, and in Table XVIII, below, these deviations will be apparent.

(a) The subject drops further and further behind time in starting; he never catches up.

(b) The subject ends his work earlier in each second, for each stroke, as the work goes on.

(c) The subject may fall back a stroke or half a stroke, if the timing signal is favorable, as in curve 6, Table XVIII, and then keep time.

(d) The variability in keeping time is comparatively high at the beginning of the work curve, falls as adjustment is made, and then rises to a very high level as the work period progresses.

TABLE XVIII

Giving the time when the contraction was started and when the relaxation ended; with deviations from the average time of start and finish. (Deviations are from the average of actual start and finish times, not from metronome time.)
The strokes measured are the $^1/_{15}$, $^4/_{15}$, $^7/_{15}$, etc.
Measurement is in seconds.

*Contraction**

No.	$^1/_{15}$	*D*	$^4/_{15}$	*D*	$^7/_{15}$	*D*	$^{10}/_{15}$	*D*	$^{13}/_{15}$	*D*	$^{15}/_{15}$	*D*
1	—.20	.022	—.32	.109	—.42	.153	—.46	.082	—.50	.184	—.45	.043
2	.264	.038	—.38	.169	—.40	.133	—.45	.072	—.46	.144	—.54	.133
3	—.24	.062	—.30	.089	—.48	.213	—.50	.122	—.50	.184	—.62	.213
4	—.43	.252	—.13	.081	—.18	.087	—.32	.058	—.26	.056	—.85	.443
5	—.10	.078	.05	.261	—.07	.197	—.24	.138	—.05	.366	—.14	.267
6	.04	.218	—.10	.111	—.05	.217	—.28	.098	—.14	.176	.15	.557
7	—.58	.402	—.30	.089	—.27	.003	—.40	.022	—.40	.084	—.40	.007
Av.	—.178	.210	—.211	.130	—.267	.143	—.378	.084	—.316	.171	—.407	.237

*Relaxation**

No.	$^1/_{15}$	*D*	$^4/_{15}$	*D*	$^7/_{15}$	*D*	$^{10}/_{15}$	*D*	$^{13}/_{15}$	*D*	$^{15}/_{15}$	*D*
1	.16	.086	.05	.095	.01	.267	.07	.054	.10	.148	.23	.040
2	.34	.266	.04	.185	.04	.127	.10	.024	.16	.088	.26	.010
3	.04	.034	.10	.045	.05	.117	.05	.074	.14	.108	.13	.140
4	.03	.044	.30	.155	.30	.133	.12	.004	.35	.102	—.14	.410
5	—.10	.174	.20	.055	.26	.093	.10	.024	.30	.052	.25	.020
6	.30	.226	.30	.155	.35	.183	.18	.056	.45	.202	.90	.630
7	—.25	.324	.11	.035	.18	.013	.25	.126	.24	.008	.26	.010
Av.	.74	.165	.145	.104	.167	.133	.124	.052	.248	.101	.27	.180

*Contraction almost always started after the signal, as shown by the minus signs. Relaxation usually completed before the second was ended and so most of the signs are plus.

3. In regard to the duration of time spent on the strokes the following facts are apparent in Table XIX.

(a) The amount of contraction time is less than the amount of relaxation time, particularly throughout the middle section of the curve.

(b) The length of both contraction and relaxation times are reduced by the end of the work curve to about one-half of what they were at the beginning.

(c) Relative variability in amount of time expended in work is comparatively high at the beginning of the

TABLE XIX

Measurements of the duration of contraction and relaxation of individual strokes at different points of the fatigue curve.
Measurement in seconds.

Contraction

No.	1/15	*D*	4/15	*D*	7/15	*D*	10/15	*D*	13/15	*D*	15/15	*D*
1	.36	.03	.24	.007	.22	.017	.22	.014	.17	.012	.15	.008
2	.21	.12	.22	.027	.20	.037	.18	.026	.14	.042	.16	.018
3	.33	.00	.25	.003	.27	.003	.18	.026	.24	.042	.10	.042
4	.35	.02	.26	.013	.20	.037	.18	.026	.20	.018	.13	.012
5	.43	.10	.28	.033	.20	.037	.20	.006	.14	.042	.14	.002
6	.30	.03	.28	.033	.30	.063	.30	.094	.20	.018	.20	.058
7	.26	.07	.25	.003	.27	.033	.24	.036	.22	.038	.10	.042
8	.40	.07	.20	.047	.24	.003	.15	.056	.15	.032	.16	.018
Av.	.33	.055	.247	.021	.237	.032	.206	.035	.182	.030	.142	.025
Rel. D		.017		.008		.014		.017		.017		.018

Relaxation

No.	1/15	*D*	4/15	*D*	7/15	*D*	10/15	*D*	13/15	*D*	15/15	*D*
1	.30	.101	.30	.076	.30	.015	.24	.048	.16	.072	.20	.032
2	.36	.041	.40	.024	.38	.065	.29	.002	.26	.028	.16	.008
3	.60	.199	.40	.024	.28	.035	.27	.018	.14	.092	.09	.078
4	.44	.039	.34	.036	.23	.085	.27	.018	.15	.082	.12	.048
5	.10	.301	.28	.096	.32	.005	.40	.112	.25	.018	.13	.038
6	.70	.299	.54	.164	.36	.045	.36	.072	.53	.298	.37	.202
7	.46	.059	.35	.026	.33	.015	.30	.012	.18	.052	.10	.068
8	.25	.151	.40	.024	.32	.005	.18	.108	.19	.042	.18	.012
Av.	.401	.147	.376	.059	.315	.034	.288	.049	.232	.085	.168	.061
Rel. D		.037		.016		.011		.017		.037		.036

Per cents which corresponding times of contraction are to the times of relaxation. (Av's. basis of figures.)

82 65 75 71 78 84

Per cent duration of strokes in the curve (Av.) to the first stroke.

Contraction

100 74 71 62 55 43

Relaxation

100 93 78 71 57 41

work curve, falls as adjustment is made, and then rises again. This variability in amount of time is greater for relaxation than for contraction.

4. In order to get some idea of the variation in contraction time of consecutive strokes, this time was measured for a section of one curve and is given here with the height of each stroke.

Contraction time	.36	.30	.25	.23	.32	.30	.20	.24	.21	.24	.23
Height	20	19.5	19.2	19	17.2	18	17.2	16.8	15	16	14

The length of contraction time is in general proportional to the height of the stroke, though there is individual variation that cannot be so accounted for. Whether the cause of quicker contraction time is greater effort and if so, how costly that effort is, or whether it be due to mechanical factors such as swinging or vibration of the weight, cannot be told from our records. Further investigation of this phenomenon is needed.

Conclusions

1. Variability in time of stroke apparently obeys the same laws as relative variability to diminishing output, as found in other sections of this study. The reason for this would apparently be that as the metronome beat is a constant factor, not slowing up with the fall of the work curve, actual difficulty in keeping time obeys the same laws as difficulty in keeping a proportional output with diminished work.

2. Variability in amount of time expended in work obeys the same laws as variability in amount of work done, as measured by length of stroke.

3. In keeping time to a set mechanism a worker tends to lose time rather than gain.

4. As the work goes on the subject spends less time working, even though keeping time to a metronome; his rest each second is longer, his stroke "smaller" in every respect.

5. The variability in the relaxation time of the individual strokes is greater than the variability in the contraction time.

6. The contraction time of the individual strokes is less than the relaxation time.

CHAPTER IV

Variations of Performance in the Course of Training

We have considered the variation of performance with practice in two ways: first, the variations in output between different working spells; second, the changes of variability within the curves themselves.

The two problems are examined separately under Part One and Part Two.

Historical

Studies of variations in performance during the course of training have been of 3 different types: (1) those that treat practice as almost synonymous to learning, finding that it leads to increase of speed, accuracy, ease of working, or quality of output; (2) those that treat of it as affecting the variability of the individual and determining the manner in which he approaches his task and organizes his responses; (3) those that have examined individual differences between subjects throughout a practice series.

There are many studies of the first type that examine the rate of progress, attempt to push the improvement all the way to the physiological limit, rid the curve of its plateaus, etc. A discussion of these will be found in the text book of Experimental Psychology by R. S. Woodworth and A. T. Poffenberger.[36]

Practice as considered under (2) above has been examined most often in regard to studies of other problems, such as the effect of drugs.

Hollingworth,[37] in a study of the correlation of specific abilities as affected by practice, finds that as practice continues, variability is reduced.

In another investigation[38] Hollingworth refers to the fatigue studies of Cattell, Thorndike, Arai, and states that the failure to reach practice limits in the work measured before

[36] Woodworth, R. S., and Poffenberger, A. T., "Text Book of Experimental Psychology," Chap. II (mimeographed edition).

[37] Hollingworth, H. L., "Correlation of Abilities as Affected by Practice," Journal of Educational Psychology, Vol. 4, 1913, pp. 404-414.

[38] Hollingworth, H. L., "Variations in Efficiency during the Working Day," Psychological Review, Vol. 21.

beginning the fatigue experiments has detracted from the conclusions of the studies.

A practice series is now accepted as a necessary part of all such experiments.

F. L. Wells,[39] in writing of practice and the work curve, introduces the third problem of individual differences. He writes:

"If, in respect to practice and the work curve, some persons show one tendency and some an opposite tendency, it is clear that no universal rule for the effect of practice on the work curve can be laid down; but it is also necessary to know whether the presence of one tendency or another, in definite degree, is a fundamental property of the individual. Different persons may vary widely about their group average, but if their differences are characteristic, they are much more important psychologically than a central tendency, which may be fairly reliable for the group, but about which the same individual may vary now in one direction, now in another. . . . That individual differences exist in the present results has already been mentioned."

Wells in a previous article[40] has shown that in an adding and cancellation test individuals maintain approximately the same relative positions before and after a moderate degree of practice.

Part One

Variations in Output between Different Working Spells

Method

A series of from 6 to 14 practice periods, depending upon the ergograph and the uniformity of output, were given each subject before the results of his work were taken for the studies previously described. Even after the main experiment was begun, certain work periods were given under the same conditions as the practice series. The results from all this work done under similar conditions were measured for number of strokes, average length of stroke, and total output or foot-pounds. These figures are here shown tabulated

[39] Wells, F. L., "Practice and the Work Curve," American Journal of Psychology, Vol. 24, 1913, pp. 35-51.

[40] Wells, F. L., "American Journal of Psychology," Vol. 23, 1912, p. 75.

and plotted serially, giving a practice curve for each one of the three factors, for each subject.

In order to have some quantitative data on practice, the totals for the first 6 work periods were taken and compared with the totals for the last 6. These comparisons are expressed in the percentages which the last 6 were of the first 6 (See Table XXII, page 62.).

Again, the total amount of work done on all machines in the first 6 work periods was taken, and the percentage found of this, to the work done in 14 periods, the total series of some subjects. This material is given in Table XXIII, page 62.

Results

1. In the tables and figures on pages 55 to 63 are shown the practice work done on the arm, weight hand, and Mosso ergographs for Subjects 2 and 4. Similar tables and graphs were made for all ten subjects for the output on all the ergographs, but as the work of Subjects 2 and 4 on the machines named may be considered typical, the other data are omitted.

Among the facts evident from an inspection of these tables are the following:

TABLE XX

Subject 2—Practice Curves.

Mosso

Practice Period	*No. of Strokes*	*Av. Length of Stroke*	*Work*
1	267	14.5	3860
2	137	16.1	2203
3	94	25.4	2396
4	257	18.5	2765
5	171	18.5	3150
6	36	47.8	1720
7	53	35.5	1883
8	43	41.6	1787
9	37	44.8	1659
10*	48	42.0	2018
11	44	40.6	1786
12	35	50.57	1770
13	27	48.25	1303
14	36	54.11	1948
15	38	49.07	1865
16	37	51.41	1902
17	27	48.03	1297
18	41	34.92	1432

*End of Practice series, as such, in the experiment.

(Continued on next page)

Arm

Practice Period	*No. of Strokes*	*Av. Length of Stroke*	*Work*
1	32	29	928
2	37	27.6	1021
3	40	21.5	858
4	36	25.9	933
5	40	23.7	949
6	41	24.8	1017
7	70	17.2	1205
8	44	28.2	1242
9	47	21.9	1031
*10**	48	19.4	929
11	52	20.00	1040
12	46	20.52	944
13	52	18.56	965
14	44	18.88	831
15	58	18.82	1092
16	48	19.26	926
17	39	21.72	847
18	36	22.22	800
19	39	19.46	759
20	51	19.70	1005
21	38	19.97	759
22	39	19.46	759
23	33	22.48	742

Weight Hand Machine

Practice Period	*Mo. of Strokes*	*Av. Length of Stroke*	*Work*
1	130	54	7021
2	255	36.5	9320
3	302	32.7	9866
4	91	56.1	5115
5	159	29	4609
6	99	37.5	3716
7	63	36.5	2300
8	51	41.37	2110
9	66	43.71	2885
*10**	76	38.55	2930
11	70	29.52	2067
12	50	46.44	2322
13	42	41.83	1757
14	45	33.75	1519
15	52	47.27	2458
16	41	43.58	1787
17	54	42.15	2276
18	41	42.17	1729

(a) Individual differences between the subjects are marked. Subject 2 produced quite variable work at the first of the practice series, remaining fairly constant in the later part of it. Subject 4 shows no marked changes in the beginning, though perhaps

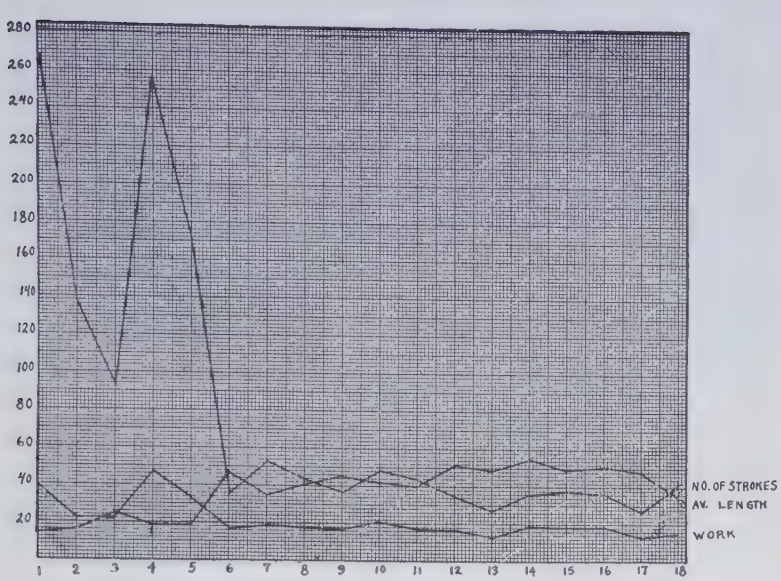

FIGURE 10
Practice Curves. Subject II. Mosso Ergograph.

there is a little more variability from one period to the next one following, throughout the whole series, than is true with Subject 2.

(b) The length of stroke is the most constant of the

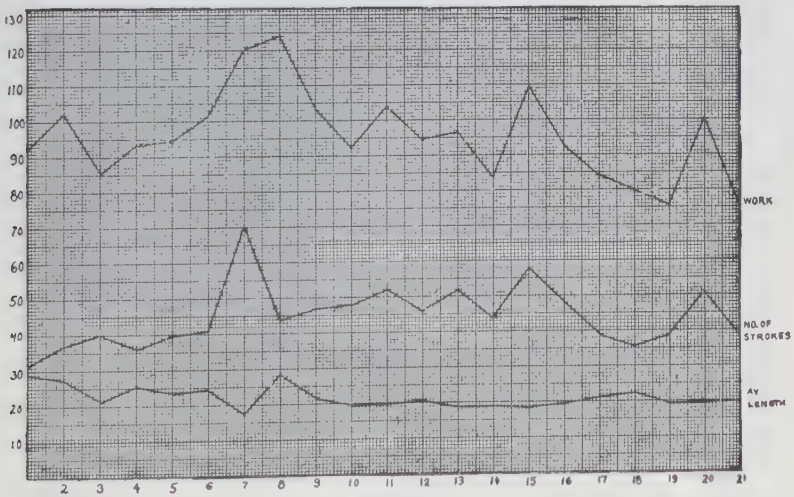

FIGURE 11
Practice Curves. Subject II. Arm Ergograph.

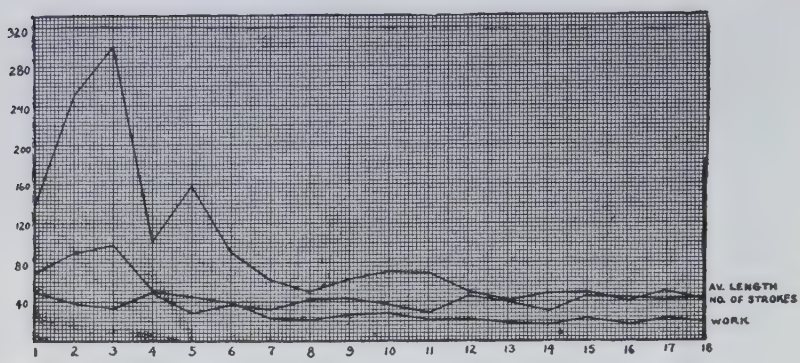

FIGURE 12
Practice Curves. Subject II. Weight Hand Ergograph.

three factors measured. This is to be expected, however, since it was the factor most evident to the subject, and upon which he worked, in nearly all cases, with knowledge.

(c) Amount of work done is more closely co-ordinated with number of strokes than with the length of stroke.

(d) All subjects tend to increase the average length of stroke, particularly with the Mosso, reduce the number of strokes, and reduce output.

TABLE XXI
Subject 4—Practice Curves.
Mosso

Practice Period	*No. of Strokes*	*Av. Length of Stroke*	*Work*
1	49	14.2	697
2	41	16.9	694
3	59	14	827
4	41	22.9	941
5	42	21.4	900
6	52	19.1	992
7	47	22.5	1057
8	49	20.6	1008
9	42	19.1	804
*10**	53	17.8	943
11	38	26.40	1003
12	34	25.44	865
13	36	28.97	1043
14	31	23.61	732
15	38	30.60	1163
16	43	23.91	1028
17	42	25.60	1075
18	38	28.68	1090
19			

(*Continued on next page*)

Arm

Practice Period	No. of Strokes	Av. Length of Stroke	Work
1	30	15.6	468
2	25	20.4	512
3	29	21.1	612
4	26	20.6	538
5	27	17.7	480
6	24	18.5	446
7	30	18.5	557
8	31	19.7	611
9	30	18.4	553
10*	27	20.9	556
11	26	20.2	525
12	30	18.43	553
13	30	19.20	576
14	29	16.24	471
15	28	16.29	456
16	30	16.10	483
17	25	20.15	504
18	35	14.94	523
19	27	17.89	483
20	31	18.90	586

Weight Hand Machine

Practice Period	No. of Strokes	Av. Length of Stroke	Work
1	55	38.8	2136
2	66	49.1	3239
3	68	38.6	2628
4	62	40.6	2515
5	61	38.5	2234
6	69	39.6	2735
7	73	36.0	2629
8	71	36.2	2571
9	53	45.8	2425
10*	71	35.12	2494
11	59	41.52	2450
12	56	38.55	2159
13	49	40.87	2003
14	60	38.30	2298
15	43	39.04	1679
16	64	42.84	2742
17	50	38.96	1948
18			

*——end of practice series, as such, in the experiment.

Two possible causes may be given in explanation of these changes: (1) The subjects may have been learning to obey directions better. The directions instructed the subjects to work as hard as they could all the time until exhausted. By learning to make longer strokes the subjects worked harder than formerly and so tired more quickly. (2) Or possibly

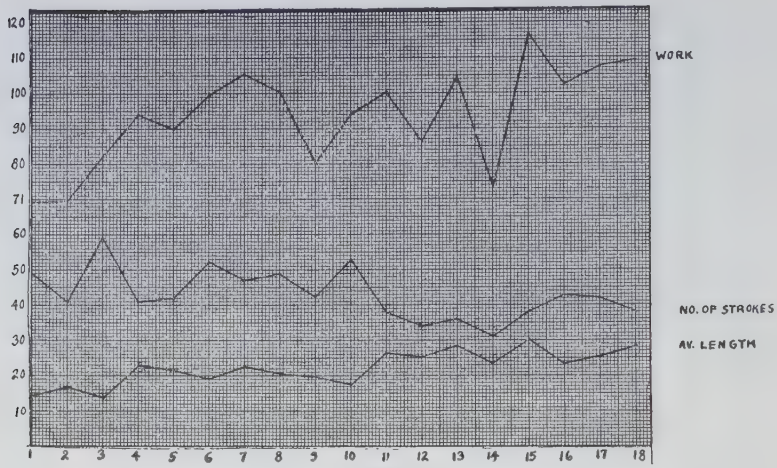

FIGURE 13
Practice Curves. Subject IV. Mosso Ergograph.

the directions had little to do with the process. It may be that with practice the subjects adjusted themselves in a more congenial fashion to the task, and that long strokes with a shorter work-spell represent this adjustment.

(e) In a number of cases the last 2 or 3 work periods were more productive than the preceding ones, showing an increase in output when the end of the experiment was in sight.

2. In order to have a more quantitative statement of changes with practice, the percentage which the output of different machines during the last 6 spells was to the output during the first 6 spells was calculated and is shown in Table XXII, page 62. The work is that of Subjects 1, 2, 3, and 4.

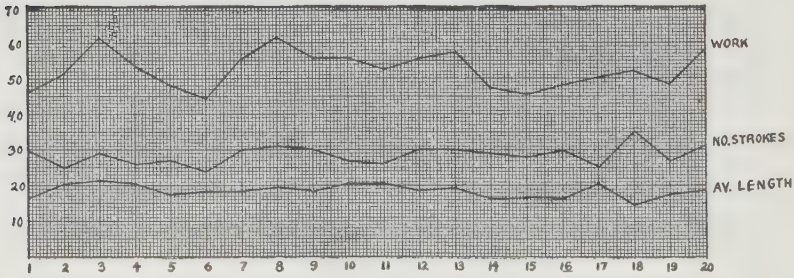

FIGURE 14
Practice Curves. Subject IV. Arm Ergograph.

In Table XXIII, page 62, the work on all the machines for each of the four subjects, totaled at the end of the first 6 periods and at the end of 14 is given, with a ratio of the first 6 to the 14.

The following statements will be found substantiated in the tables.

(a) Again, decided individual differences are apparent. Subjects 1, 3, and 4 appear to be more consistent workers than 2, though they do less actual work than he does. (See the AD's in Table XXIII and

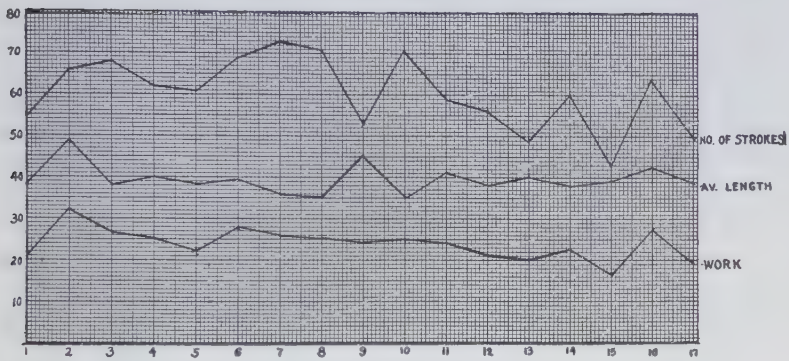

FIGURE 15
Practice Series. Subject IV. Weight Hand Ergograph.

observe per cent reduction with practice in Tables XXIII and XXIV.

(b) Subject 2 is the most variable worker of the group from every angle in which he has been studied. By the analysis of the curve in quarters (Chapter I) he is found more variable than the other men studied here.

(c) Subject 2 did more work rowing by 123 per cent than his nearest competitor of this group. He did more work with the weight-hand machine than any one else at rowing. He did almost twice as much work in 14 periods as Subject 4, and as much in the first 6 periods as either 1 or 4 did in 14; but the percentage fall of the last 6 to the first 6 is 149 per cent that of the fall of output of Subject 4, and 127 per cent of Subject 1. He did 55 per cent of the total 14 periods work in the first 6 periods as against 45, 46, and 47 per cent by the other workers.

TABLE XXII

Percentage Output of Last Six Work Periods to the First Six.

Machine	*No. of strokes*	*Av. Length*	*Work*
Subject 1			
Mosso	87	107	93
Arm	84	95	80
Wt. Hand	67	135	88
Rowing	62	79	51
Leg	68	108	85
Av.	74	108	79
AD	10	17	11
Subject 2			
Mosso	22	202	54
Arm	104	91	84
Wt. Hand	26	102	29
Rowing	109	82	96
Leg	47	106	85
Av.	62	115	624
AD	36	34	26
Subject 3			
Mosso	49	148	80
Arm	101	85	85
Wt. Hand	105	96	99
Rowing	77	83	59
Leg	106	96	99
Av.	88	102	844
AD	20	19	12
Subject 4			
Mosso	80	149	121
Arm	109	91	99
Wt. Hand	84	97	83
Rowing	78	92	81
Leg	88	102	83
Av.	88	106	932
AD	09	17	13

TABLE XXIII

Total Work Done on All Machines at the End of the First Six periods, and at the End of Fourteen, with Ratio.

Subject	*Total of 6*	*Total of 14*	*Ratio*
1	48006	101825	47%
2	108035	194531	55
3	60021	130213	46
4	44767	99308	45

Part Two

Changes of Variability within the Curves, during Practice

Method

Variability figures for the four quarters of the work curve calculated in Chapter I are here arranged for the work-spells in a serial manner. This is not a complete practice study, however, because it begins with the starting of the main experiment and shows practice changes affecting data used in the main portion of this study.

Its value rests in a comparison with Part One of this chapter, where it is seen that changes between work-spells continue in a certain fashion to the very end of the experiment; and it is also of value in casting light on the reliability of the results of Chapter I, in so far as changes with practice would invalidate our conclusions.

Results

Arm ergograph curves, made with the 24-pound weight pulled at a 2-second rhythm with a 10-minute rest between the two spells of the work period provide us with the data. The work for all 10 subjects for a series of 8 practice periods was averaged and is given here in Table XXIV, page 64. Figures for the quarters of the curve were averaged separately.

An examination of this data leads us to the conclusion that the variability of the work, at different stages of the work curve, did not make any appreciable change during the experiment.

This indicates that our conclusions in Chapter I could not have been invalidated in any way by a practice effect.

In contrast to this uniformity of the degree of variability at different stages of the work curve throughout practice is the constant lessening of output, etc., and general lengthening in stroke, found in comparing different work-spells for the same practice period (See Part One, Chapter IV.).

We may perhaps conclude that the degree of variability within the curve is more permanently characteristic of the individual and of the working conditions than are the other

TABLE XXIV

All Subjects, Variability with Practice. Calculated Separately for the Four Quarters of the Curve.

Quarter		*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8 Practice*
1	Av.	.027	.043	.029	.029	.032	.027	.028	.031
	QD	.010	.021	.004	.009	.010	.004	.009	.014
2	Av.	.066	.061	.045	.048	.046	.054	.046	.063
	QD	.035	.035	.021	.029	.025	.028	.035	.036
3	Av.	.142	.136	.164	.179	.163	.157	.168	.161
	QD	.023	.036	.033	.073	.045	.059	.082	.073
4	Av.	.327	.380	.442	.353	.383	.345	.353	.358
	QD	.130	.104	.155	.094	.200	.114	.139	.076

factors mentioned, such as total output and the length of stroke made.

But before this conclusion may be considered proved, the study of variability within the curve must be carried back to the very beginning of practice.

Conclusions

1. As practice continues there is an adjustment to the task, represented here by longer strokes. This practiced adjustment does not ensure that more work will be done.
   (a) If this practice effect be explained as an improvement in following directions, as it may be, it would indicate that directions to work "as hard as you can till exhausted," would lead, at times, to poor methods of work and reduced output.
   (b) If the practice effect be explained as a gradual adjustment to a more congenial effort at the task, it would indicate that long strokes with an intensive application to the task for a short time might be more congenial than prolonged work at a less intense level.
2. The fact that the part of his output most evident to the worker is length of stroke, increased with practice, suggests that if the worker knew at all times exactly how much his output was, he might strive to keep it up to a uniform level or increase it.
3. Individual differences are in some cases striking, and

emphasize the conclusions in Chapter I that workers should be tested for variability as well as for output.

4. If subjects know when their experiment will be concluded, there tends to be an increased output at the end, more work being accomplished during the last work-spells than during earlier ones.

5. Conclusions for Chapter I are free from any possibilities of error due to practice changes.

6. The degree of variability within the curve appears to be comparatively constant with the same individual, under particular working conditions.

CHAPTER V

General Conclusions

1. Relative variability of output increases with fatigue. This is found true in Chapter I, Part One, for total variability which includes the fall in the curve; in Chapter I, Part Two, it is found true for incidental variations; in Chapter III the relative variability of several curves from an average curve is found to increase constantly with fatigue.

2. Variability of performance in beginning work can be affected by any one of a number of causes: the fact of having done previous work, the length of the ensuing rest period, the rhythm of work, the load, etc. It seems possible that variability might be made a measure of the effect of any one of these factors, by keeping all the others constant. Chapter I.

3. The variability curve for output in any one period is determined somewhat by the weight pulled. Chapter I. The deviations of several curves at different points in those curves are very definitely affected by the weights pulled. Chapter III.

4. Individual differences in variability are both striking and important. These are found in comparing different parts of the individual work curves, and in comparing different work periods. Chapters I and IV.

5. Individual differences appear to be greatest at the low output end of the work curve.

6. The increase of variability with fatigue is due rather to a loss of control than directly to the exhaustion of muscle.

7. Deviations among several work curves at different points in those curves are greater toward the low output end of the curves, but less again near the exhaustion point. Chapter III.

8. Difficulty in keeping time to a metronome increases with continued work. The tendency is to lose time. Chapter III.

9. The amount of time expended on work in the individual strokes is proportional to the amount of work done as measured by length of stroke. As work goes on the subject spends less time in working even though keeping time to a metro-

nome; his rest each second is longer, his stroke "smaller" in every respect. Chapter III.

10. The contraction time of individual strokes is less than the relaxation time. Chapter III.

11. Subjects do not always learn, with practice, to work in the most efficient way. This may be due to (a) a closer following of directions, such as those given in this experiment, that lead to more intensive but less efficient work; or (b) a gradual adjustment to a more congenial effort.

12. If the date for finishing the experiment is known to the subjects they tend to do more work in the last few work-spells than in the preceding ones. Chapter IV.

13. The degree of variability within the work curve appears to remain comparatively constant for a given individual, under particular working conditions. Chapter IV.

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VITA

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